

Leveraging Technology-Enabled Revealed Preference Information by Sequentially Pricing Multiple Products

John Aloysius

Cary Deck

Amy Farmer

University of Arkansas

December 2011

Technological advances enable sellers to price discriminate based upon a customer's revealed purchasing intentions. E-tailers can track items in "shopping carts" and RFID tags enable retailers to do the same in bricks and mortar stores. In order to leverage this information, it is important to understand how this new visibility impacts pricing and market outcomes. We propose a model in which a seller sets prices for Goods A and B, allowing for the possibility of sequentially revising the price for good B if the buyer reveals a preference for Good A by making an initial purchase decision. We derive comparative statics results for the prices of products that have super-additive or sub-additive values, and also for the associated profits. We also run simulations for a range of distributions of buyer values, to compare sequential pricing with mixed bundling. The results indicate that IT enabled sequential pricing can increase profits relative to mixed bundling or pure components pricing for substitute goods due to a reduction of intra-seller competition. We also consider the case of goods with positively or negatively correlated values and find that when sellers can condition the second good's price on the buyer's decision to purchase the first good, sequential pricing increases profits when customer's values for the goods are highly positively correlated.

Key Words: Sequential Pricing, Price Discrimination, Online Shopping Carts, RFID Tags, Mixed Bundling, Complements/Substitutes, Correlated Product Values

1. Introduction

Advances in information technology provide retailers, both online and in stores, with an almost omniscient ability to monitor customer behavior and consequently to use strategic pricing in a manner that was unimaginable until now. As evidenced by several recent NY Times articles, increasingly firms are attempting to leverage information that is gained by technology ranging from tracking electronic coupons (Clifford 2010a) to tracking credit card purchases (Martin 2010) to tracking customer in-store behavior (Clifford 2010b) in order to price discriminate. While Information Systems researchers have studied technologically enabled price setting processes and issues in online Business to Customer (B2C) auctions (e.g., Bapna, Goes, and Gupta 2003a, 2003b), and in Business to Business (B2B) transactions (e.g., Bandyopadhyay, Barron, and Chaturvedi 2005, Bichler, Shabalin, and Pikovsky 2009), there has been little research in the IS literature on exploiting information gained from monitoring customer behavior within a shopping experience for the purpose of price discrimination, perhaps due to the relatively recent adoption of technologies such as RFID.¹ This paper explores the theoretical implications of firms tracking customers while visiting a store or a website and leveraging this information to drive prices.

Traditionally, when a shopper walks into a large department store searching for a new outfit prices are already posted on the shelves. A buyer observes a variety of shirts, selects one, and then moves on to look at pants. In this familiar story, sellers are making a simultaneous pricing decision. One way in which sellers have attempted to exploit information on the underlying distribution of preferences among goods is by selling a collection of items in a bundle; even so the prices are still set *ex-ante*. For example, the department store could sell an outfit rather than pants and shirts.² However, advances in IT are such that pricing decisions for multiple products can be sequential rather than simultaneous. Now, a seller is able to identify which shirt a customer selects (or simply picks up for a moment) before setting the pants prices the shopper will observe. Since product selection reveals information about the buyer's tastes and preferences, sellers are better able to estimate a buyer's willingness to purchase

¹ One exception is Jiang, et al (2009), which develops an algorithm for online pricing of consumer selected bundles.

² The term pure bundling refers to offering only the bundle, whereas mixed bundling refers to offering both the individual items and the bundle simultaneously. Only offering the items separately is termed pure components pricing. While not normally explicit, the assumption in both cases is that the buyer is considering the purchases simultaneously.

other items and tailor prices accordingly. Thus, a seller could raise the price for pants coordinating with the selected shirt, perhaps by offering a smaller coupon for the preferred pants.³

Consider the situation where two people, High (H) and Low (L), each value two products A and B . Suppose, $V_A^H = V_B^H = V$ and $V_A^L = V_B^L = v$ where $V > 2v > 0$ and the value of consuming the bundle containing both products is $V_A^i + V_B^i$ for $i = H, L$. For simplicity, assume that the marginal cost of each product is 0. Under pure components, the maximum attainable profit is $2V$, reaped by selling both products at a price of V to the high valued buyer, H . The seller could obtain $4v$ from selling both items to both buyers at v , but by assumption $4v < 2V$. Under mixed bundling the firm can again only earn $2V$, all from person H , who will again purchase both goods. The inability of mixed bundling to increase profits is due to the fact that no discount can be offered for the bundle that would result in L buying the bundle while H bought the items separately or vice versa. However, under sequential pricing with discrimination, the seller can obtain a profit of $2V+v$. Assuming that the decision to purchase A is made first, the seller can set the price of good A at V . H will purchase A and L will not. The seller can then set the good B price at V for those who purchased A and set the good B price at v for those who did not. In this case both H and L would buy good B but at different prices. The preceding illustration motivates the current research – what is the optimal pricing strategy for sellers who can monitor a customer’s initial purchase decision? The problem of conditional sequential pricing is one of first being able to identify the order in which prices are observed, being able to identify the customer’s action and then exploiting this information. The practice of tailoring prices to individual customers based on willingness to pay is called *targeted pricing* (Zhang 2009). The practice of selling an additional product or service to an existing customer is commonly called *cross-selling* (Kamakura, Kossar, and Wedel 2004). This research analyzes targeted pricing strategies available to a technologically enabled all-seeing retailer who has visibility into the initial product preferences of a customer and uses this information to cross- sell.

The above example also illustrates two key aspects of the seller’s problem: the degree to which buyers are forward looking and connect the two pricing decisions and the ability for products to be presented

³ Of course, the seller could extract information about the value of different shirts based upon the pants considered if the shopper considered the products in the other order. The point is not about being able to control the order in which shoppers observe different products, but rather exploiting the sequential nature of shopping for multiple products. For the sake of this paper it does not matter if the presentation order is exogenous or endogenous as long as the seller can identify that an order exists either by tracking the buyer or by construction of the shopping experience. In bricks and mortar retail, the flow of people through the store is carefully arranged. For example, retailers leave impulse items for the checkout stand. In online markets shoppers have to actively negotiate websites to observe prices and the seller can track this history. The optimal order in which to present goods in the face of individually tailored marketing promotions is an open empirical question.

sequentially. Some pricing literature (e.g., Aviv and Pazgal 2008, Fudenberg and Villas-Boas 2005) has assumed strategic customers when the buyers might be anticipating price fluctuations for the product that is under consideration (e.g., this shirt may go on sale in the future). The strategic behavior employed in these papers is quite simple from the customer point of view. If the consumer recognizes that the price of a durable item might fall over time, they may wait to make a purchase, a behavior that the seller must consider when choosing current and planned future prices. This is significantly less sophisticated than a consumer recognizing that their entire basket of purchases might be analyzed and the future price of all potential complements and substitutes might be affected. Further, a strategic forward looking consumer in this context would have to be capable of not only recognizing that fact, but then determining the optimal manner in which a seller might alter prices given those choices and the distribution of preferences of all other customers in the market, and then in some fashion make optimal current period buying decisions to manipulate the seller's pricing path to their advantage. This is the model type we will be investigating in this paper, and modeling strategic buyers in this context requires a significant leap in terms of behavioral sophistication from the modeling of strategic consumers that currently exists in the literature, and existing literature on buyer sophistication suggests that an assumption of the sort mentioned above would be inappropriate. There is considerable evidence that humans are boundedly rational, and thus some researchers question the appropriateness of modeling buyers as being fully rational (Ellison 2006). Others recognize that rationality may not hold particularly when pricing decisions involves add-ons such as cartridges for a printer (Gabaix and Laibson 2006). Behavioral research shows that decision makers typically frame problems narrowly rather than broadly, deciding about local options without considering all alternatives (March 1994). People are content to find a set of sufficient conditions to solve a problem rather than the most efficient set of conditions. March (1994) also lists problem decomposition as a coping strategy: people attempt to decompose large problems (e.g., how to make two sequential purchase decisions) into their component parts. Such problem decomposition in the context of retail buying decisions may result in myopic decision behavior, since buyers focus on a product purchase without considering strategies that will impact future prices of other goods. Retailers already use technologies that also enable sequential pricing to make product recommendations, which implies that buyers are often unfamiliar with other items a seller has to offer. Assuming that buyers have formed expectations of prices for these yet to be recommended items when considering the initial item seems unrealistic. While the example above deals with two goods, many retailers carry a large number of SKUs (Stock Keeping Units) and expecting a buyer to understand how

each possible combination of purchases could affect future promotions also seems unrealistic.⁴ Therefore, our focus is primarily on the case of myopic buyers for this initial study.

While it is not new for sellers to target consumers based on their purchase history via personalized and customized marketing strategies (See Arora et. al. 2008 for a survey), new technologies are making it possible to observe prior and current purchases in real time as a customer is making a choice. Sellers have access to vast databases that can be mined to determine underlying relationships in buyer values across goods (Ulph and Vulkan 2003; see Kamakura et al. 2004, Ghose and Sundararajan 2006 for examples). Sellers routinely record the contents of every shopping basket sold. If purchases are made with credit or debit cards or some other form of identification such as a frequent buyer cards, a customer's shopping history within and across retailers can be compiled. Until recently however, such customer identification has been costly (Vulkan 2003). Techniques like collaborative filtering and content filtering enable websites such as Amazon.com to provide recommendations to specific customers for other products based upon the information in such databases. Ansari et al. (2000) point out that among other sources, a customer's preferences or choices is information that can be used to make recommendations, and knowing that one person is more likely than someone else to enjoy an item also suggests that the person is willing to pay more.⁵

Ulph and Vulkan (2003) refer to the ability to better tailor prices and thereby extract greater surplus caused by the wave of personalization in e-commerce as the *enhanced surplus extraction effect*. As previously described, technology enabled real-time monitoring of customers provides retailers with the ability to identify customers based upon revealed preferences and as a result, to better optimize pricing decisions taking advantage of both customer self-selection and enhanced surplus extraction. Identification of which items a buyer has observed is straightforward in online markets where buyers place items in electronic shopping carts and cookies allow a shopper to be monitored. For example, when purchasing airline tickets through an online service, shoppers are often shown ads for hotels and attractions near their destination. It is easy to display a different set of ads and promotional offers to

⁴ Shell (2009, p. 153) claims that customers only know the prices of 1-2% of the hundreds of thousands of SKUs carried by Walmart.

⁵ Ayres (2007) gives many examples of firms that currently engage in such practices, ranging from the most visible examples of supermarkets printing coupons based upon purchases to those less visible such as Harrah's casinos. Harrah's records real-time data on players winning or losing, and in combination with demographic information, uses this information to offer complementary promotional benefits to players who lose more than a critical threshold amount. In this way they avert these players leaving with a negative experience from their visit to the casino.

those who have booked a Saturday night stay (or purchased a child's ticket). Can the same level of monitoring occur in physical stores? The answer is yes, due to ubiquitous computing technologies (Acquisti 2006). Currently, RFID (radio frequency identification) technology is being used to monitor which products buyers in bricks and mortar stores have in their physical shopping carts. This technology is being employed primarily for theft detection, but other forward looking applications are being explored by industry and academia (ADT Reference Library 2009, Cromhout et al. 2008). Retailers, such as the Dillard's department store among others, have introduced item level tagging in pilot stores and are planning expansion of the program. Sam's Club, Walmart's retail warehouse club division, expanding on its previous pallet level tagging mandate, recently introduced an item level tagging mandate for its suppliers, requiring that they tag all items shipped to 22 distribution centers by 2010 (Weier 2008). The chain is poised to unveil a new RFID enabled customer checkout system that will considerably reduce transactions costs and improve inventory control. In physical stores, current practices are such that all buyers observe the same quoted prices; however, the use of new smart shopping carts can transfer the price display from the shelf to the shopper's cart allowing each shopper to receive a unique price or coupon.⁶ Alternatively, individualized coupons can be sent to each shopper's phone. According to a December 17, 2008 article in the *New York Times* by Bob Tedeschi, this technology is already being employed by companies such as Cellfire and 8Coupons to offer coupons from retailers such as Sears and Kroger's.

Technology is also making it easier to dynamically change prices in brick and mortar stores (which of course is already easy to do in online stores). Retailers ranging from supermarkets (PressReleasePoint 2010) to fashion apparel stores (Happich 2010) are pilot testing e-paper price labels that can be instantaneously be updated by a central system. These initiatives are currently pursued to cut costs (Epaper central 2009) resulting from price changes including labels, product packaging, and labor. However, once the technology is in place using that infrastructure to facilitate the type of cross-selling and targeted marketing that is the subject of the current research is inevitable. Thus, while price discrimination strategies have been both studied and implemented previously, emerging technologies make much more refined targeting significantly more accurate and cost effective. Much prior research has focused on the profitability of using market segmentation techniques (such as targeted coupons) in a competitive market. That work is not focused on optimal pricing strategies across goods, but rather on

⁶ United States Patent 5729697 is for just such a device. The carts are also touted as way to monitor the health content of a shopper's purchases. Even without such technology, buyers may still pay different prices depending on the coupons they have, some of which could have been mailed specifically to them.

the overall cost effectiveness of using data regarding demographics and purchase history to make targeted offers. Another line of research examines monopoly pricing of a durable good where the price today affects purchases both today and in the future. Neither of these veins of work examines the precise pricing strategy of separate products whose values to the consumer may or may not be related, and how the distribution of joint valuations of consumers translates into optimal sequential pricing. This paper extends prior research in this arena to optimal monopoly pricing over time *across* products that may be substitutes or complements given that it is possible to observe purchase choices in the first period and not only use that to price the other product in the second period but also to choose optimal pricing in the current period.

The results presented in this paper suggest that sequential pricing with discrimination outperforms mixed bundling when the goods are close substitutes. Sequential pricing even without discrimination is more profitable than simultaneous pricing of pure components when the goods are weak substitutes. The rest of this paper is organized as follows. Section 2 reviews research on pricing relevant to the current research framework. Section 3 outlines an analytical framework that models sequential pricing decisions faced by a monopolist while Section 4 provides the results of numerical simulation. In Section 5 we consider the case of forward looking rational buyers. Section 6 summarizes the results and offers comment.

2. Literature Review

Research on sequential pricing and exploiting the underlying relationship among the goods is surprisingly sparse given the rapid proliferation of technologies that enable retailers to gather information on likely purchases that could be used to set prices on those candidate goods in order to maximize expected profit. Zhang (2009) observes that in practice targeted pricing frequently does not fit any of Pigou's (2006) three different forms of price discrimination, going beyond the standard understanding of price discrimination. We suggest that in addition, implementing the proliferating schemes that emerge in practice requires information that is assumed to be known by standard theoretical models, but is not known in practice. Moreover, the use of such data, until recently, has been very costly; however, Rossi, McCulloch and Allenby (1996) show that targeted pricing achieved through the use of demographic and purchase history data can raise profits. There are numerous studies of dynamic pricing of goods, although these are confined to single goods and do not consider cross category effects on other goods. Cope (2006) presents dynamic strategies for maximizing revenue in internet retail by actively learning customers' demand responses to price. Zhang and Krishnamurthi

(2004) provide a decision support system of micro-level promotions in an internet shopping environment, that provides recommendations as to when, how much, and to whom to give price promotions. Jiang et. al (2011) examine optimal pricing strategies when internet customers create their own bundles one product at a time. Although their model has a sequential nature to it as the bundle is constructed chronologically, that paper does not use purchasing information in one period to update information regarding preferences.

There are several examples of sellers using customer behavior to infer preferences, and using that information either to drive revenues or for customer relationship management. Montgomery et al. (2004) show how clickstream data about the sequence of pages or path navigated by web buyers can be used to infer users' goals and future path. There is a literature on behavior-based price discrimination (for a survey see Fudenberg and Villas-Boas 2006) in which firms use information about consumers' previous purchases to offer different prices and/or products to consumers with different purchase histories. Empirical data shows that even the information contained in observing one historic purchase occasion by a customer boosts net target couponing revenue by 50% (Rossi et al. 1996). Automotive retailers can use information on which online infomediary customers use in order to market segment and price discriminate (Viswanathan et al 2007). Mulhern and Leone (1991) specifically address multi-product pricing and develop a framework for retail pricing and promotion policies. Using empirical data, they estimate the influence of regular and promotional prices on sales of substitute and complementary goods, and thus demonstrate the effectiveness of price promotions as a means of exploiting interdependencies in demand among retail products. Another stream of work uses purchase history to identify customers of one's own firm as well as those of a competitor. With that information, this line of research considers the benefits of poaching a competitor's customers versus targeting and retaining one's own loyal customers. (See Fudenberg and Tirole 2000, Zhang and Wedel 2009, Shaffer and Zhang 1995, 2000, 2002, Chen and Iyer 2002 and Choudhary, Ghose, Mukhopadhyay and Rajan 2005 for examples).

In the absence of competition, researchers have examined price discriminating monopolists who price a single product over time. These works are focused on cannibalization of one's own product when either pricing a durable good over time (Fudenberg and Villas-Boas 2005) or introducing a product of differing quality (Moorthy and Png 1992, Dhebar 1994). McAfee and Vincent (1995) examine this in an auction format. Fudenberg and Tirole (200) do consider a monopolist selling two distinct goods in which consumers are modeled to have relative preferences for one good or the other and will purchase only

one. As such, they cannot examine substitutability/complementarity of products nor can they consider correlated valuations across products, both of which we do in this paper. This adds complexity to the pricing decision in period 1, knowing that attracting or losing customers will affect their valuation of the product offered in the second period. Moreover, the consumers are assumed to value both goods so highly that the monopolist always prefers to make a sale; the result is that the model does not allow a tradeoff between extracting surplus from a consumer and losing a sale. In their model consumers who chose to purchase A will receive the same price in period 2, thereby losing no surplus from having been revealed.

Aside from these few exceptions, the literature on selling multiple products has been primarily focused on bundling by monopolists.⁷ Bundling has been shown to be an effective price discrimination tool even when the consumer's willingness to pay for each good is independent of the value of the other good and the value of the bundle is the sum of the values of the components (Adams and Yellen 1976).

Customers with a high degree of asymmetry in product valuations will buy an individual product that they favor, while customers with more symmetric valuations will buy the bundle.⁸ Venkatesh and Kamakura (2003) present an analytical model of contingent valuations and find that the degree of complementarity or substitutability in conjunction with marginal cost levels determines whether products should be sold as pure components, pure bundles, or mixed bundles. They also find that typically, complements and substitutes should be priced higher than independently valued products. Subramaniam and Venkatesh (2009) and Leszczyc and Haubl (2010) find that in an auction it is best to bundle strong complements, but otherwise separate auctions are preferred. Nettesine, Savin, and Xiao (2006) present a stochastic dynamic program for analyzing the selection of complementary products.

Better ability to predict preferences has been shown to potentially reduce price competition (Chen et al. 2001). Acquisti and Varian (2005) show analytically that it is optimal to price so as to distinguish between high-value and low-value customers. There is empirical evidence that competing firms have

⁷ There are a few studies of price bundling in competitive markets. McAfee, McMillan, and Whinston (1989) extend their monopoly results to a duopoly and show that independent pricing can never be a Nash equilibrium when the reservation prices for the single goods are independent. Chen (1997) analyzes a situation in which firms compete in a duopoly for a single product and the firms also produce other products under conditions of perfect competition. Bundling as a product differentiation device proves to be an equilibrium strategy for one or both of the firms. Aloysius, Deck and Farmer (2010) report behavioral experiments where firms engage in bundling while competing for informed customers and maintaining monopoly power over uninformed customers.

⁸ Schmalensee (1984) finds similar results in a model with continuous (bivariate normal) valuations. McAfee, McMillan and Whinston (1989) provide conditions under which such bundle pricing is optimal. Hanson and Martin (1990) show how to compute optimal bundle prices using a mixed integer linear program, while Jiang, et al (2009) develop an algorithm for pricing bundles incrementally constructed by consumers.

been able to price discriminate profitably by charging different prices across consumer segments (Basenko et al 2003). Moon and Russell (2008) develop a product recommendation model based on the principle that customer preference similarity stemming from prior purchase behavior is a key element in predicting current purchase. These studies exploit customer revealed preference for a good in order to set prices for future purchases of that good. The current research extends the issues explored in previous research to study how buyer revealed preferences inferred from initial purchase decisions of one good, can be used to set optimal prices for purchases of other goods. One advantage of this new mode of target marketing is that information on revealed preference can be used in the same online or in-store visit. Furthermore, it is not dependent on identifying customers in order to access their buying history.

3. A Model of Sequential Pricing with Myopic Consumers

We begin with the simple assumption of a seller facing a pricing decision in two sequentially ordered markets.⁹ For our purposes it only matters that the seller can determine the order in which the products were considered, not that the seller can control this ordering. The optimal pricing strategy depends upon whether the seller can use information regarding the consumer's decision in the market for the first good when setting the price for the second good. We restrict our analysis in the current research to the situation in which the consumer considers buying a single unit of each good (e.g., two different books or a shirt and pants outfit). This situation would arise in the case of goods that have some degree of substitutability (subadditivity in joint consumption) but offer distinct values. While two books may be close substitutes if they cover similar topics from the same perspective, it is unlikely that someone would buy two copies of one of the books rather than one of each. We first consider the case in which the monopolist cannot use such information, perhaps due to technological constraints, fear of consumer backlash or government action. This is followed with an analysis of the case in which the monopolist is able to price discriminate in the market for the second good.

Before considering these two cases, it is useful to note that sequential pricing in the absence of price discrimination is substantively different from simultaneous pricing. The sequential pricing problem is one in which the seller recognizes the impact of the price of good A on the purchase of good B. By recognizing the behavioral response at the second stage (good B decision) to the outcome in the first

⁹ The buyer is assumed to only consider purchases from this seller effectively making the seller a monopolist in this market. A lack of comparison shopping could be due to actual market monopolization, high search or switching costs, or a preference for the particular seller. The order of the markets need not be controlled so long as the seller can identify the order before determining what price the buyer will observe for the second item.

stage (good A decision), the seller will consider those results in expectation when pricing good A. Consumer behavior differs in that the sequential problem does not provide the consumer with full information when making a choice. Rather, the consumer in this model is assumed to be myopic, choosing to purchase A solely on its price relative to value and then a choice regarding B will occur subsequently. Given that consumer behavior is different due to the timing effects, the pricing strategy is also different. As such, it is important to fully model the case of sequential pricing absent price discrimination.

We begin with the general market set-up and then consider the each case in turn. Assume a market exists for two products denoted A and B , and a consumer has a choice first (without loss of generality) to buy A followed by a choice to buy B in sequence. Let the consumer's value for A be distributed $V_A \sim f_A(V_A)$. Then the consumer will buy A iff the price of good A is such that $P_A \leq V_A$. Similarly, the consumer's independent value for B follows the distribution $V_B \sim f_B(V_B)$.¹⁰ Following Venkatesh and Kamakura (2003), a consumer's joint value from purchasing both A and B is denoted $V_{AB} = (1 + \theta)(V_A + V_B)$, where θ represents complementarity if $\theta > 0$, and substitutability if $\theta < 0$. A consumer who chooses not to purchase A will buy B iff the price of good B is such that $P_B \leq V_B$. However, if A was purchased, then the joint value becomes relevant and the consumer will buy B iff $P_B < (1 + \theta)(V_A + V_B) - V_A$ which can be rewritten as $V_B > \frac{P_B - \theta V_A}{1 + \theta}$. As is standard in the literature (e.g. Adams and Yellen 1976 and Venkatesh and Kamakura 2003) consumer will purchase at most one unit of each item, and the monopolist produces each item at constant marginal costs of C_A and C_B respectively.

Given this framework, now consider a seller's price setting problem.

Case 1: Sequential Pricing without Price Discrimination

When the seller sets P_B , it is as if there is no information concerning the decision to buy A . Rather, the seller will know the probability that A will have been purchased, conditional on the price of A and the distribution of preferences. Similarly, the seller sets the price of A knowing the probability A will be bought and therefore, how that probability will affect the subsequent purchase of B . Therefore, the solution requires backwards induction. In other words, we must first consider the optimal price of B

¹⁰ In the next section, we provide the results of numerical simulations where the independence assumption is relaxed.

conditional on a price of A and the corresponding probability that A was purchased. Then, given that optimal response to a price of A, we can determine the optimal price for good A.

Stage 2: Price of B

The seller will maximize expected profit with respect to the price of B conditional on a price of A and the distributions over preferences. Profits from the sale of good B are $P_B - C_B$ if a sale is made and 0 otherwise. The seller maximizes equation (1).

$$\begin{aligned} \max_{P_B} E\Pi(P_B|P_A) = & (P_B - C_B) \int_{P_B}^{\infty} \int_0^{P_A} f_A(V_A) f_B(V_B) dV_A dV_B + \\ & (P_B - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f_A(V_A) f_B(V_B) dV_B dV_A \end{aligned} \quad (1)$$

The first term is profit from those who buy B but not A and the second term is profit from those who buy both B and A. Differentiating (1) with respect to P_B to determine the first order condition and solving for $P_B^* = f(P_A)$ gives an optimal response function based upon the choice in stage 1.

Stage 1: Price of A

Given P_B^* , the seller must choose the optimal P_A in stage 1 by maximizing expected profit for the sum of both stages, recognizing that the choice of P_B depends on P_A . In stage 1, the seller maximizes equation (2).

$$\begin{aligned} \max_{P_A} E\Pi(P_A) = & (P_A - C_A) \int_{P_A}^{\infty} \int_0^{\frac{P_B^* - \theta V_A}{1 + \theta}} f_A(V_A) f_B(V_B) dV_B dV_A \\ & + (P_B^* - C_B) \int_{P_B^*}^{\infty} \int_0^{P_A} f_A(V_A) f_B(V_B) dV_A dV_B \\ & + (P_A - C_A + P_B^* - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B^* - \theta V_A}{1 + \theta}}^{\infty} f_A(V_A) f_B(V_B) dV_B dV_A \end{aligned} \quad (2)$$

where the first term is profit from those who buy A only, the second term is profit from those who buy B only, and the third term is the profit from those who buy both. The general solution is derived by finding the first order condition of (2) with respect to P_A , solving this first order condition for P_A^* and then calculating P_B^* .

The first order conditions derived from the maximization of (1) and (2) can be found in Appendix A. Using those first order conditions to characterize the equilibrium, we perform comparative statics to analyze the effect of the price of A on the subsequent price of B as well as the impact of θ on prices and

profits. While closed form solutions cannot be derived, some insights into these comparative statics for general functions can be found. Specifically we have the two following propositions, the proofs of which can be found in Appendix A.

Proposition 1: When a monopolist engages in sequential pricing without discrimination and the goods are substitutes (complements) the price of good B is decreasing (increasing) in the price of good A. That is $\theta < 0$, $\frac{dP_B}{dP_A} > 0$ and when $\theta > 0$, $\frac{dP_B}{dP_A} < 0$.

Proposition 2: When a monopolist engages in sequential pricing without discrimination, expected profit is increasing in θ . In other words, as intuition would suggest, as goods become more complementary, sequential pricing results in greater expected profits.

Case 2: Pricing with Price Discrimination

In this case the seller will know when setting the price of B whether the consumer has purchased A or not. Formally, the decision is to choose either $P_B | (q_A=0)$ or $P_B | (q_A=1)$ where $q_A = 0$ if A was not purchased and $q_A = 1$ otherwise. In other words, the seller selects a state contingent price for good B.

Stage 2: Price of B if $q_A=0$, i.e. $V_A < P_A$

Beginning with the case in which buyer's values for A and B are independent, in the event that A is not purchased, the seller's problem is to maximize

$$E\Pi(P_B | (q_A = 0)) = (P_B - C_B) \int_{P_B}^{\infty} f_B(V) dV_A \quad (3)$$

Taking the first order condition of (3) and solving yields $P_B^* | (q_A=0)$.

Stage 2: Price of B if $q_A=1$, i.e. $V_A \geq P_A$

In this case the seller considers the joint valuation of both products when pricing B. In other words, $V_{AB} = (1 + \theta)(V_B + V_A)$. Thus, the marginal value of B is $(1 + \theta)(V_B + V_A) - V_A$ and the consumer will buy B iff $\frac{P_B - \theta V_A}{1 + \theta} \leq V_B$.

Given this information, the seller chooses P_B to maximize equation (4).

$$\max_{P_B} E\Pi(P_B | (q_A = 1)) = (P_B - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f(V_B) f(V_A | V_A \geq P_A) dV_B dV_A \quad (4)$$

Taking the first order condition of (4) and solving yields $P_B^* | (q_A=1)$.

Stage 1: The Price of A

We now need to solve for P_A given what will occur in stage 2. Specifically we need to know $E[\pi]$ when $q_A=0$ and $q_A=1$. Plugging the solution for $P_B^* | (q_A=0)$ and $P_B^* | (q_A=1)$ into (3) and (4) respectively gives the expected profit in each state. The seller will choose the price of A to maximize total expected profit over both stages, knowing both the probability that good A will be purchased at a given price and the resulting expected profits in stage 2 based on the follow-up price of good B. That is the seller will maximize (5).

$$E\pi(P_A | P_{B_0}^*, P_{B_1}^*) = F(P_A)P_{B_0}^* (1 - F(P_{B_0}^*)) + [1 - F(P_A)]P_{B_1}^* \int_{P_A}^{\infty} \int_{\frac{P_{B_1}^* - \theta V_A}{1+\theta}}^{\infty} f(V_B)f(V_A | V_A > P_A) dV_B dV_A \quad (5)$$

The general first order condition fully characterizing optimal prices for sequential pricing with discrimination can be found in Appendix B as can the proofs of the following propositions.

Proposition 3: When a monopolist engages in sequential pricing with discrimination, the optimal price of good B for customers who buy A is decreasing in the price of good A regardless of the degree of complementarity. That is when A was purchased, $\frac{dP_B | (q=1)}{dP_A} < 0$ regardless of the sign of θ .

Proposition 4: When a monopolist engages in sequential pricing with discrimination, expected profit is increasing in θ . In other words, as intuition would suggest, as goods become more complementary, sequential pricing results in greater expected profits.

4. Sequential Pricing in Markets with Specific Distributions of Buyer Values

The general problem discussed in section 3 does not have a closed form solution making it difficult to draw inferences about sequential pricing or make comparisons with other pricing strategies. Therefore we consider sequential pricing under specific distributional assumptions. First we consider the case of buyer values being uniformly distributed. This assumption is commonly used in the literature (e.g. Adams and Yellen 1976 and Venkatesh and Kamakura 2003) and results in analytical solutions for the seller's problem (as shown in section 4.1). Under the assumption of a uniform distribution, we are able to compare the profitability of sequential pricing to pure components pricing and mixed bundling (in section 4.2). For robustness, we also consider profits under the assumption that buyer values have a symmetric triangular density and a symmetric bimodal density with peaks at the extreme values (in section 4.2).¹¹ Finally, we consider profitability when buyer values for the two goods are correlated (in

¹¹ The joint distribution for the symmetric triangular distribution resembles a truncated pyramid, but its density is a 90° rotation of the pyramid density (see Kettler 2005). The joint bimodal distribution resembles a cube with a

section 4.3), essentially replacing the $f_A(V_A)f_B(V_B)$ with $f(V_A, V_B)$ in section 3. Profits with the triangular and bimodal distributions and in the case of correlated values are based upon numerical simulations.

4.1 Sequential Pricing under a Uniform Distribution

In this subsection we derive solutions for sequential pricing with and without discrimination under the assumption that buyer values are independently and uniformly distributed. That is $f_A(V_A) \sim U[0,100]$ and $f_B(V_B) \sim U[0,100]$.

Case 1: Sequential Pricing without Price Discrimination

In this case (1) can be rewritten

$$\max_{P_B} E\Pi(P_B|P_A) = (P_B - C_B) \int_{P_B}^{100} \int_0^{P_A} \frac{1}{100^2} dV_A dV_B + (P_B - C_B) \int_{P_A}^{100} \int_{\frac{P_B - \theta V_A}{1+\theta}}^{100} \frac{1}{100^2} dV_B dV_A \quad (1')$$

After integrating and simplifying (1') the problem becomes

$$\max_{P_B} E\Pi(P_B|P_A) = \frac{(P_B - C_B)}{100^2} \left[100^2 - P_A P_B + \frac{P_B}{1+\theta} (P_A - 100) + \frac{\theta}{1+\theta} \left(\frac{100^2 - P_A^2}{2} \right) \right] \quad (1'')$$

Differentiating (1'') with respect to P_B and simplifying, the first order condition yields

$$P_B^* = \frac{-C_B(P_A\theta - 100) - \frac{100^2}{2}(2+3\theta) + \frac{\theta P_A^2}{2}}{-2(P_A\theta + 100)} \quad (6)$$

Performing a similar exercise using a uniform distribution, (2) can be rewritten as

$$\begin{aligned} \max_{P_A} E\Pi(P_A) &= (P_A - C_A) \int_{P_A}^{100} \int_0^{\frac{P_B^* - \theta V_A}{1+\theta}} \frac{1}{100^2} dV_B dV_A + \\ & (P_B^* - C_B) \int_{P_B^*}^{100} \int_0^{P_A} \frac{1}{100^2} dV_A dV_B + (P_A - C_A + P_B^* - C_B) \int_{P_A}^{100} \int_{\frac{P_B^* - \theta V_A}{1+\theta}}^{100} \frac{1}{100^2} dV_B dV_A \end{aligned} \quad (2')$$

Simplifying (2') yields (2'')

$$\max_{P_A} E\Pi(P_A) = \frac{(P_A - C_A)}{100} (100 - P_A) + \frac{(P_B - C_B)}{100^2} \left[100^2 - P_A P_B^* + \frac{P_A P_B^* - 100 P_B^*}{1+\theta} + \frac{\theta(100^2 - P_A^2)}{2(1+\theta)} \right] \quad (2'')$$

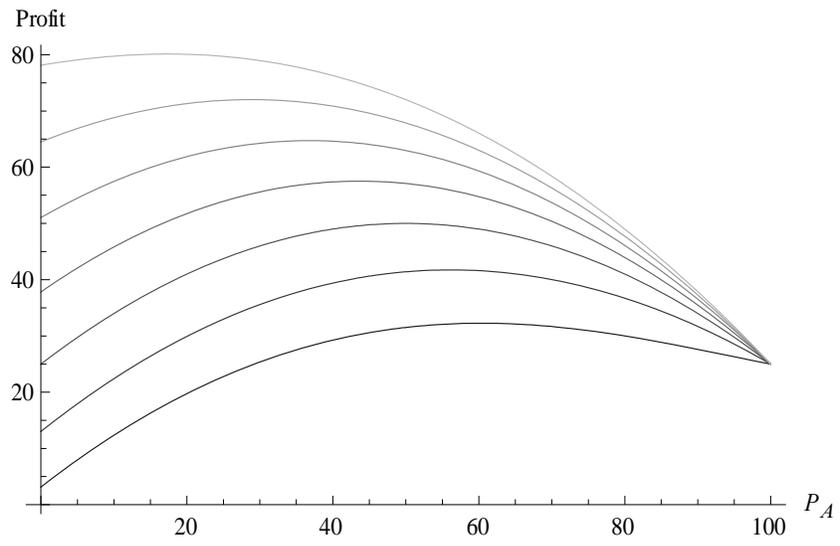
downward facing rotated pyramid removed from the top. For ease of exposition we drop the word symmetric when discussing the triangular and bimodal distributions, but that should be understood throughout the paper.

where P_B^* is found in equation (6). Solving the first order condition for (2') we need to differentiate with respect to P_A given that P_B is a function of P_A . The first order condition is given by the following fourth order polynomial in P_A :

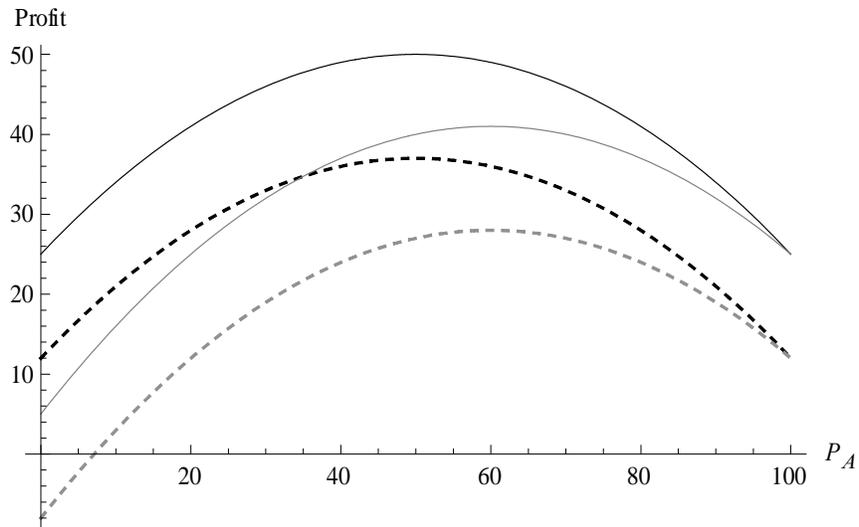
$$0 = \{3P_A^4 \theta^3 + 8P_A^3 \theta^2 [-350 + (-400 + C_B) \theta] + 4P_A^2 \theta [-160000 + 400 (-325 + C_A + C_B) \theta + (25000 + 400C_A + C_B^2) \theta^2] + 800P_A [-40000 + 100 (-100 + 4C_A + 4C_B) \theta + (25000 + 400C_A + C_B^2) \theta^2] + 20000 [8000 + (6000 + C_B^2) \theta - 6000\theta^2 - 4500\theta^3 + 80C_A (1 + \theta)]\} [160000 (1 + \theta)(100 + P_A \theta)^2]^{-1}.$$

Although quite cumbersome, real solutions can be found using Mathematica. Panel A in Figure 1 shows profit as a function of P_A for values of θ varying from -0.5 (lower dark curve) to 1 (upper light gray curve) where P_B satisfies (6) and assuming that $C_A = C_B = 0$. It is clear from this figure that profits increase in θ and that the optimal price of good A decreases with θ . Panel B of Figure 1 considers changes in C_A and C_B assuming that $\theta = 0$. Black curves indicate that $C_A = 0$ while gray lines indicate that $C_A = 20$, and solid curves indicate that $C_B = 0$ while dashed lines indicate that $C_B = 20$. From this figure, one can see that an increase in C_A leads to lower profits and an increase in the optimal P_A , while an increase in C_B leads to lower profits but does not change the optimal P_A . C_B 's lack of impact on P_A^* is due to the assumption that $\theta = 0$ and is not generally true. Together, these two figures demonstrate the smoothness of the profit function over the relevant range for the various parameters. Panel C of Figure 1 graphs the maximum profit, P_A^* , and P_B^* as functions of θ when $C_A = C_B = 0$. Once again we notice that profits are increasing in θ . The logic of why P_A^* falls with an increase in θ is obvious from this graph; by charging a lower price for good A the firm can greatly increase the price of good B. Finally, we note that when $C_A = C_B = \theta = 0$, $P_A^* = 50$, $P_B^* = 50$, which are precisely the optimal prices in the two independent markets taken separately.

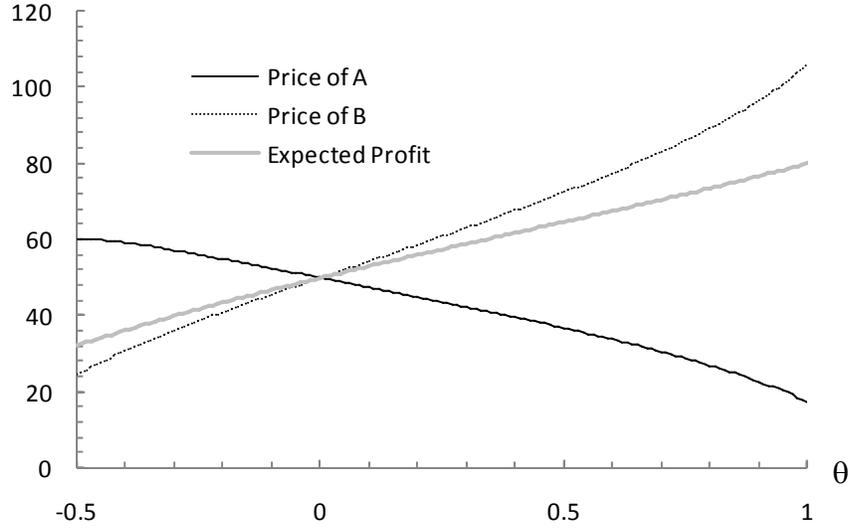
Figure 1. Numerical Results for Sequential Pricing without Discrimination



Panel A: Optimal Profit as a function of P_A for θ varying from -0.5 (black line) to 1 (light gray line)



Panel B: Optimal Profit as a function of P_A for C_A varying from 0 (black) to 20 (dashed) and C_B varying from 0 (solid) to 20 (dashed).



Panel C : Optimal Prices and Profits as a function of θ .

Case 2: Sequential Pricing with Price Discrimination

Stage 2: Price of B if $q_A=0$, i.e. $V_A \leq P_A$

For the uniform distribution, equation (3) simplifies to (3').

$$(P_B - C_B) \int_{P_B}^{100} \frac{1}{100} dV_B = \left(\frac{P_B - C_B}{100} \right) (100 - P_B) \quad (3')$$

Maximizing (3') with respect to P_B and solving yields $P_B^*(q_A = 0) = \frac{100 + C_B}{2}$, the familiar monopoly solution.

Stage 2: Price of B if $q_A=1$, i.e. $V_A \geq P_A$

Under the assumption of the uniform distribution, equation (4) can be rewritten as (4').

$$\begin{aligned} \max_{P_B} E[\Pi(P_B | (q_A = 1))] &= (P_B - C_B) \int_{P_A}^{100} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{100} \frac{1}{100} \frac{1}{100 - P_A} dV_B dV_A \\ &= \frac{P_B - C_B}{100(100 - P_A)} \left(100^2 - \frac{100P_B}{1 + \theta} + \frac{100^2\theta}{2(1 + \theta)} - 100P_A + \frac{P_B P_A}{1 + \theta} - \frac{P_A^2\theta}{2(1 + \theta)} \right) \quad (4') \end{aligned}$$

Taking the first order condition of (4') and solving for the price of B yields

$P_B^* = \frac{100^2(1+3\theta/2) - 100P_A(1+\theta) - \theta P_A^2/2 + (100 - P_A)C_B}{200 - 2P_A}$. Note that once again when $\theta=0$ we get the standard monopoly solution of $P_B^* = \frac{100+C_B}{2}$.

Stage 1: The Price of A

Recall that $P_B^*(q_A = 0) = \frac{100+C_B}{2}$ for the uniform case. Computing the resulting profit at the second stage yields $E\Pi_B(q_A = 0) = (P_B - C_B) \int_{\frac{100+C_B}{2}}^{100} \frac{1}{100} dV_B$ which can be simplified to $25 - \frac{C_B}{2} + \frac{C_B^2}{4(100)}$.

When $q_A=1$, $P_B^*(q_A = 1) = \frac{100^2(1+3\theta/2) - 100P_A(1+\theta) - \theta P_A^2/2 + (100 - P_A)C_B}{200 - 2P_A}$ which yields a corresponding second stage expected profit of $E\Pi_B(q_A = 1) = \frac{[200 - 2C_B + (300 + P_A)\theta]^2}{1600(1+\theta)}$.

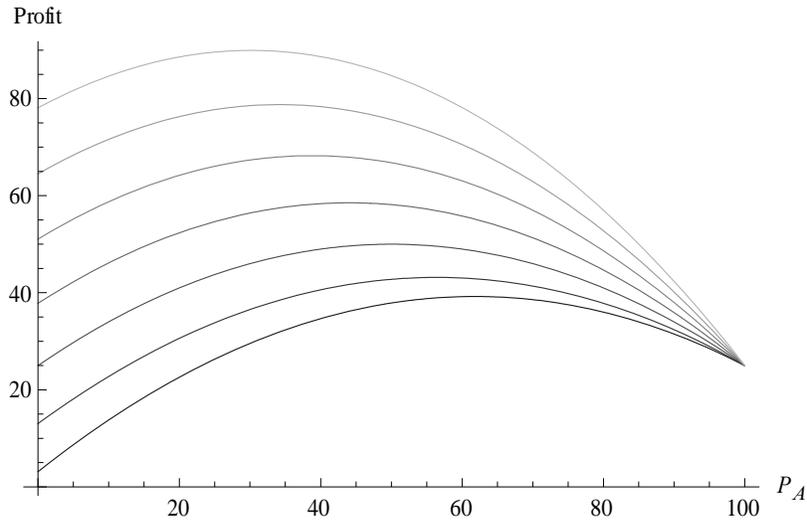
Therefore, the first stage profit as a function of P_A can be written as

$$E\Pi = (P_A - C_A) \int_{P_A}^{100} \frac{1}{100} dV_A + \left(25 - \frac{C_B}{2} - \frac{C_B^2}{400}\right) \int_0^{P_A} \frac{1}{100} dV_A + \frac{[200 - 2C_B + (300 + P_A)\theta]^2}{1600(1+\theta)} \int_{P_A}^{100} \frac{1}{100} dV_A$$

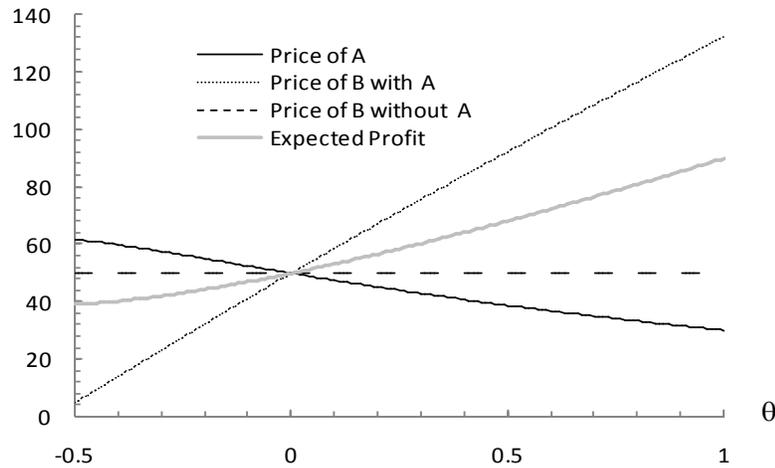
The first order condition is $\{-3P_A^2\theta^2 + 8P_A(-400 + \theta(-500 + C_B)) - 125\theta^2\} + 4[400C_A(1 + \theta) - C_B^2(2 + \theta) - 2500(-16 - 12\theta + 3\theta^2)] [16000(1 + \theta)]^{-1} = 0$. Again the real solutions can be found using Mathematica.

Panel A in Figure 2 shows profit as a function of P_A for values of θ varying from -0.5 (black) to 1 (light gray) assuming that $P_B(q_A = 0)$ and $P_B(q_A = 1)$ are set optimally and that $C_A = C_B = 0$. As we found in the case without price discrimination, profits increase in θ while the optimal price of good A decreases with θ . A panel similar to Panel B in Figure 1 is not presented here because when $\theta = 0$, and the value

Figure 2. Numerical Results for Sequential Pricing with Discrimination Assuming Zero Marginal Cost



Panel A: Optimal Profit as a function of P_A for θ varying from -0.5 (black line) to 1 (light gray line)



Panel B: Optimal Prices and Profits as a function of θ .

for the two goods are independently distributed, the ability to conditionally price the second good has no effect and thus the two graphs are identical. Panel B in Figure 2 shows the maximum profit, P_A^* , $P_B^* | (q_a=0)$ and $P_B^* | (q_a=1)$ as functions of θ when individual item values are independent and $C_A = C_B = 0$. From the figure it is clear that P_A^* falls with an increase in θ (as in the no discrimination case) while $P_B^* | (q_a=1)$ increases in θ and $P_B^* | (q_a=0) = 50$ irrespective of θ . The intuition for P_A^* falling is straightforward; by lowering the price of good A more people will purchase A and the distribution of marginal values for good B will increase. The increase is greater the larger is θ , and these higher values result in a higher optimal price of good B for those who purchased good A. The distribution of marginal

values for good B for those consumers who do not purchase good A remains uniform on $[0,100]$ and hence the optimal price of good B for these customers remains at 50. We also note that when $C_A = C_B = \theta = 0$, $P_A^* = P_B^* | (q_a=0) = P_B^* | (q_a=1) = 50$, as in the standard monopoly setting.

4.2 Profit Comparisons between Sequential Pricing, Bundling and Pure Components

In this subsection we compare the profitability of sequential pricing with mixed bundling and pure components pricing under three assumptions regarding the distribution of buyer values. Specifically, we consider the case of buyer values that are uniformly and independently distributed as in the previous subsection. We also consider the case in which buyer values are independently distributed according to a triangular distribution and a bimodal distribution.¹²

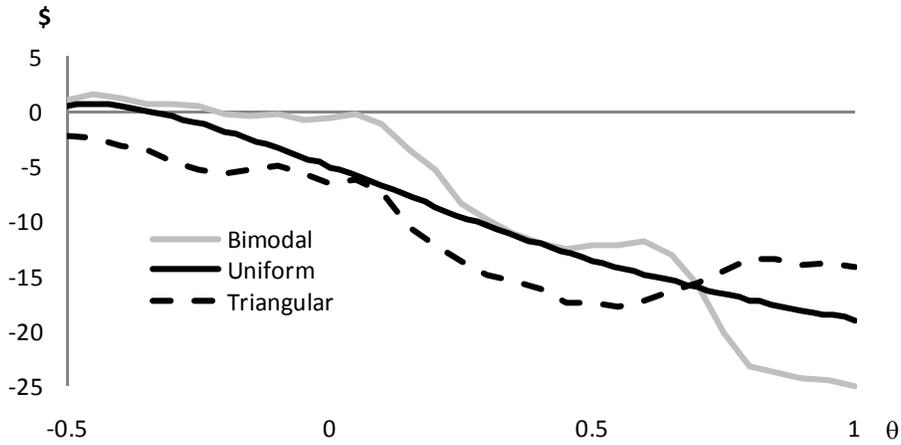
Components pricing can be thought of as a special case of mixed bundling where the bundle price is simply the sum of the individual prices in a similar way that sequential pricing without discrimination is a special case of sequential pricing with discrimination where the seller sets the same price irrespective of the good A purchase decision. Therefore, mixed bundling must be at least as profitable as components pricing and sequential pricing with discrimination must be at least as profitable as sequential pricing without discrimination. Hence, we focus on the deference between sequential pricing without discrimination and pure components pricing and the difference between sequential pricing with discrimination and mixed bundling. For simplicity we assume that marginal costs are 0.¹³

Panel A of Figure 3 shows the profit difference between sequential pricing with discrimination and mixed bundling for the uniform distribution. From the figure, sequential pricing with discrimination is more profitable than mixed bundling when the goods are “close” substitutes (i.e. θ is extremely small). The intuition for sequential pricing with discrimination being more profitable than mixed bundling when

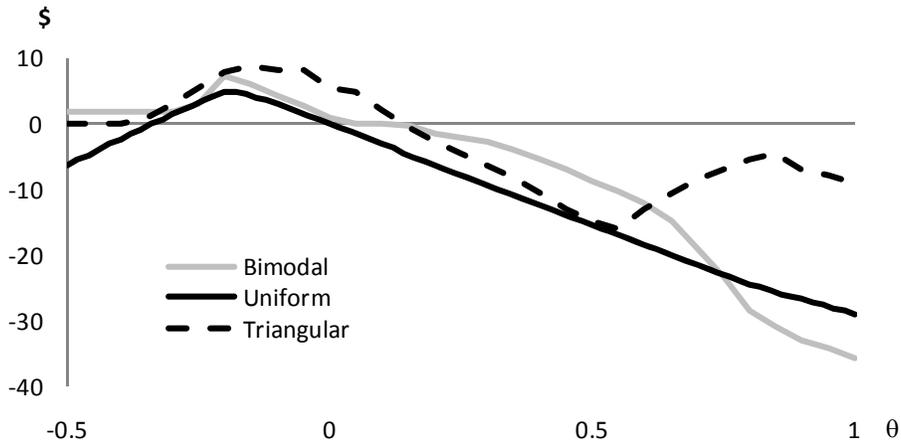
¹² Technically the uniform distribution is a discrete uniform distribution where each integer from 0 to 100 is realized with a $1/101$ chance. The triangular distribution is also discrete over 0 to 100 where the probability that $X \leq 50$ is drawn is $p(X) = (X+1)/T$ and the probability that $X > 50$ is drawn is $p(X) = (101-X)/T$ where $T = 51^2$. The bimodal distribution is also discrete over 0 to 100 where the probability that $X \leq 50$ is drawn is $p(X) = (51-X)/T$ and the probability that $X > 50$ is drawn is $p(X) = (X-49)/T$ where $T = 51 \times 52 - 1$. For both distributions T is set such that $\sum_{X=0}^{100} p(X) = 1$. Due to the added complexity of pdfs for the triangular and bimodal distribution, optimal profit in these cases are estimated by simulating 10,000,000 buyers for each integer pricing strategy. For this computation we relied upon a super computing cluster with 157 compute nodes, each with dual quad-core Xeon E5430 processors, 2x6MB cache, 2.66GHz, 1333FSB. Even with this computational power, some simulations took days to solve.

¹³ It is not possible to make general profit comparisons between pricing strategies irrespective of the assumed cost structure. For example, the firms might experience increasing return to scale in an individual item, but have large multiplicative causing firms to prefer offering a single product.

Figure 3. Numerical Comparison of Profits by Distributional Assumption and Degree of Value Additivity Assuming Marginal Cost



Panel A. Profit with Sequential Pricing with Discrimination – Profit with Mixed Bundling



Panel B. Profit with Sequential Pricing without Discrimination – Profit with Pure Components Pricing

the goods are close substitutes is that the marginal value for the second good is small relative to the case of complements. In our research setting, sequential pricing essentially avoids intra-seller competition for the two goods whereas bundling attempts to capture the marginal value. When the goods are complements (or even weak substitutes) the reduction in internal competition is not enough to offset the gains from capturing the marginal value of a second purchase, but when the goods are highly substitutable, bundling captures little value from the second product whereas sequential pricing

avoids the intra-seller competition. Panel A of Figure 3 also shows the profit difference under the bimodal and triangular distributions. The general pattern is similar in all three cases suggesting that the results are robust to the distributional form; however, sequential pricing with discrimination is only more profitable than bundling for two of the three distributions considered.

Panel B of Figure 3 shows the difference in profit between sequential pricing without discrimination and pure components pricing for all three distributional assumptions. Again, the pattern is similar for the three distributions. The general result is that sequential pricing without discrimination outperforms components pricing when the goods are weak substitutes (i.e. θ is slightly below 0).

4.3 Profit Comparisons Without the Assumption of Independent Values

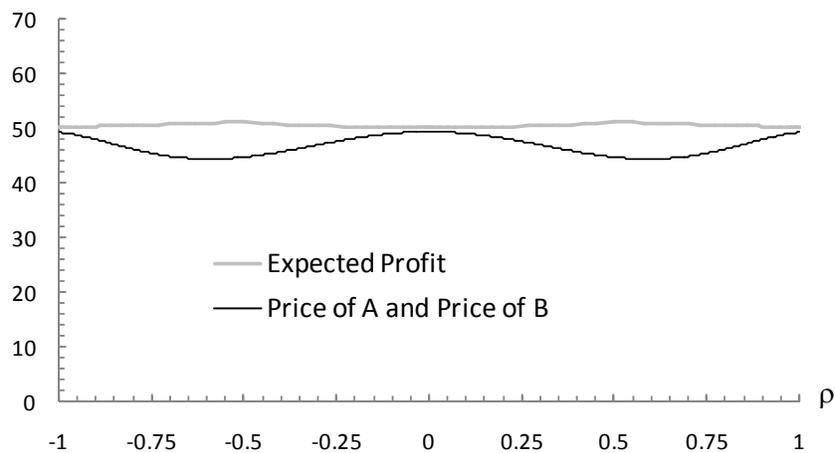
Thus far, we have assumed that buyer values for the two goods are independent. In this subsection we report the results of numerical simulations where this assumption is relaxed. First, it is important to realize that the complements/substitutes relationship is distinct from the correlation between the values of the two goods. For example, two books on a related topic can be substitutes or complements depending on the overlap in their content, but people who dislike the topic will likely have a low value for both, while people who like the topic will likely have a high value for both.

To explore how correlation, ρ , may impact profits with sequential pricing, we calculate optimal behavior in a continuum of distributions with varying correlations holding $\theta = 0$. Again, we also assume that marginal costs are 0. We choose to focus on distributions that are generated by removing opposing corners from the square domain of $[0,100] \times [0,100]$.¹⁴ The base case assumes values for each good are uniform before the corners are removed, but we also consider the case where values are triangular and bimodal. These distributions have the advantage of being directly comparable to the results described above for changes in θ . This procedure has also been used in related behavioral research by Aloysius, Deck, and Farmer (2010) to explore goods with correlated values. Once corners are removed the marginal distributions of values are changed, but for expositional simplicity we refer to each case by the pre-corner removal name and acknowledge the change with quotation marks.

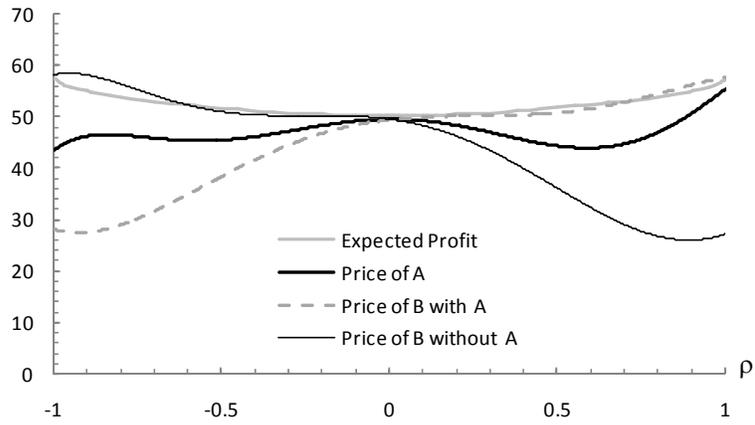
¹⁴ A distribution with negative correlation can be achieved by starting with $[0,100] \times [0,100]$ and removing all (V_A, V_B) pairs such that $|V_A + V_B - 100| > r$ for $r \in [0,100]$. When $r = 0$ we have $\rho = -1$ and the domain is $V_A + V_B = 100$ and when $r = 100$ we have $\rho = 0$ and the domain is the original square. The relationship between ρ and r is monotonic but not linear. Similarly, a distribution with a positive correlation can be achieved by starting with $[0,100] \times [0,100]$ and removing all (V_A, V_B) pairs such that $|V_A - V_B| > 100 - r$ for $r \in [0,100]$. When $r = 0$ we have $\rho = 0$ and the domain is the original square and when $r = 100$ we have $\rho = 1$ and the domain is $V_A = V_B$.

To determine optimal prices in this environment, we rely upon simulations. Specifically, for each level of ρ we simulate the expected profit associated with a given set of prices and then iterate over possible prices. Panel A of Figure 4 plots optimal prices and profits as a function of ρ , for the “uniform” distribution when sellers can sequentially price, but cannot price discriminate. Without the ability to price discriminate and no change in the distribution of marginal values of good B due to purchasing good A since $\theta = 0$, sellers set the same prices for goods A and B. When only a few potential customers are removed from the square (i.e. $|\rho|$ is small so only the most extreme combinations are eliminated) a monopolist lowers its price due to the elimination of some high value customers. The opportunity cost of lowering the price is smaller since the lower price is being charged to fewer remaining customers. However, as more and more high and low valued customers are removed (as $|\rho|$ increases) the ability to lower price and serve new customers is reduced, and the result is that monopolists begin raising their prices beyond a threshold value of $|\rho| \approx 0.58$.

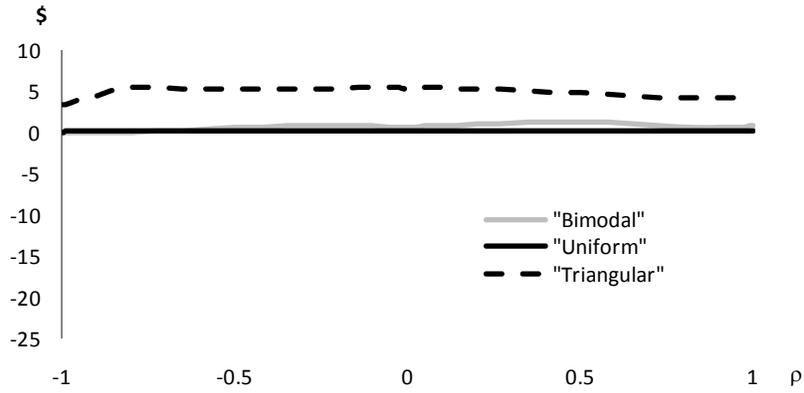
Figure 4. Numerical Results for Buyers with Correlated Values Assuming Zero Marginal Cost



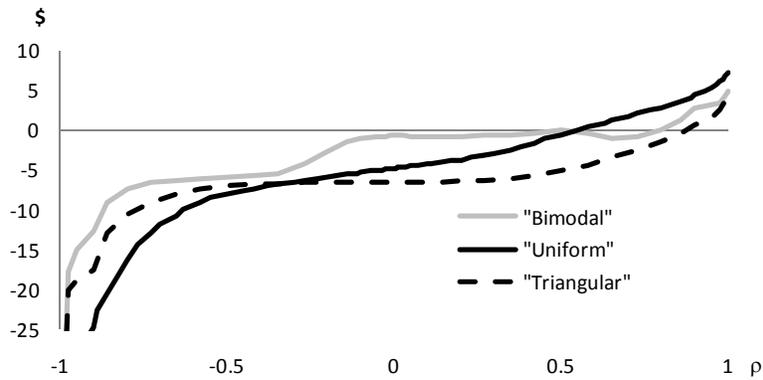
Panel A : Optimal Prices and Profits as a function of ρ for Sequential Pricing without Discrimination with the “Uniform” Distribution



Panel B : Optimal Prices and Profits as a function of ρ for Sequential Pricing with Discrimination with the "Uniform" Distribution



Panel C : Profit with Sequential Pricing without Discrimination – Profit with Pure Components as a function of ρ



Panel D. Profit with Sequential Pricing with Discrimination – Profit with Mixed Bundling as a function of ρ

Panel B of Figure 4, shows the optimal prices and expected profit as ρ varies when sellers can sequentially price and discriminate for the “uniform” distribution. Intuitively, when values are positively correlated, those who buy A are quoted a higher price for good B than those who don’t while the opposite is true when values are negatively correlated. As in the no discrimination case, optimal prices are not monotonic because the loss of high valued customers can be offset by increasing sales with lower prices up to a point. It is also worth noting that expected profit is increasing in $|\rho|$.¹⁵ Further the good B price difference is generally increasing in $|\rho|$.¹⁵ Finally, the price of good A is generally increasing in ρ , although not monotonically for the reasons described above. This upward trend is due to the strategic value of using P_A to identify the buyers with high values of good B.

Panels C and D show the profit difference between sequential pricing without discrimination and components pricing and between sequential pricing with discrimination and bundling, respectively, as a function of ρ for the “uniform”, “triangular”, and “bimodal” distributions. The most interesting result is that when buyer values are highly positively correlated, sequential pricing with discrimination is more profitable than mixed bundling for all three distributions, suggesting that this finding is fairly robust.¹⁶ Sequential pricing without discrimination yields similar profit to components pricing for the “uniform” and “bimodal” distributions and produces greater profits for the “triangular” distribution. For all three distributions, the difference does not substantively depend on ρ , which is intuitive since the inability to price discriminate means firms face the same marginal distribution of values as a seller using components pricing does.

5. Sequential Pricing under Alternative Assumptions of Consumer Behavior

5.1 Customer Reneging

Our analysis in section 4 assumes that buyers are constrained by their original decision to purchase A or not. While this assumption may be valid if the purchases are made on separate shopping trips and the customer can be tracked through time, it may not be appropriate when a customer is considering the two purchases on the same shopping trip. In such a situation, the customer often has the option to reconsider the purchase of A after observing the price of B. That is, a customer who had originally chosen not to buy A could decide to buy A after observing the price of B or a customer who had

¹⁵ Profits and $|P_B^*(q_a=0) - P_B^*(q_a=1)|$ are nearly symmetric in $|\rho|$. The slight asymmetry may be due to the discrete nature of the numerical estimation.

¹⁶ This is consistent with the motivating example discussed the introduction and revisited in the next section.

originally decided to buy A could decide not to after observing the price of B . How would the ability to *renege* impact the profitability of sequential pricing? The answer depends upon whether or not the firm is engaging in price discrimination or not and if so, what price the seller charges when the shopper's original A decision is reversed. Thus, there are three separate cases to consider.

If the seller is engaging in sequential pricing without discrimination, then regardless of the original A decision, the shopper will be presented with P_A and P_B at the second stage and can choose to buy A only, B only, both A and B , or nothing. This is theoretically identical to pure components pricing.¹⁷

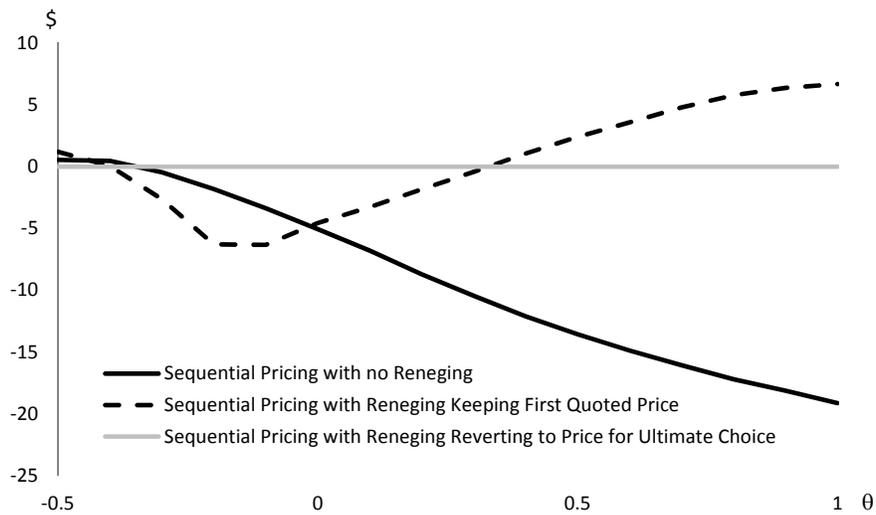
The two remaining cases focus on sequential pricing with discrimination, but differ as to whether or not a purchase decision change by the buyer triggers a price change to the seller. First, consider the situation where the seller continues to charge the price of B that the shopper initially observes. That is, if a shopper originally indicated an intent to buy A they would face a price of $P_B|(q_A=1)$ for B even if the shopper subsequently decided not to buy A . Similarly, the shopper that originally indicated she would not buy A would face $P_B|(q_A=0)$ for B even if she subsequently changed her mind and bought A . This approach may be appealing to sellers in that it avoids changing prices to a specific consumer, which can create negative backlash by eroding the buyers' trust in the seller (Kannan and Kopalle 2001). To explore this case we again rely upon computational methods to compare pricing mechanisms.¹⁸ For simplicity we focus on the case in which buyer values for each good are assumed to be independently and uniformly distributed. Figure 5 shows the profitability of sequential pricing with discrimination relative to mixed bundling when consumers can renege on the A decision, but continue to face the first quoted price of B . For comparison, Figure 5 also shows the difference between sequential pricing with discrimination when there is no renege (the model developed in the previous section) and mixed bundling. As is evident from the figure, both forms of sequential pricing outperform mixed bundling for low values of θ , when the goods are close substitutes. That is, the ability of customers to renege when renege does not trigger a price change does not substantially impact the profitability of sequential

¹⁷ Pure components and sequential pricing without discrimination may differ psychologically. There is evidence that people compartmentalize choices because of cognitive limitations (see Payne and Bettman 2001 for a discussion and review of the literature). If they do, then the likelihood of shoppers reconsidering one decision as they continue shopping may depend on the degree to which consumers perceive the products as being related, something that intuitively seems more likely for extreme values of θ (or ρ). Thus, it becomes an important empirical question if sequential pricing can outperform pure components pricing for goods that are weak substitutes.

¹⁸ One could also incorporate renege into the theoretical framework described in the previous section. However, given that the base model is already complex and lacks simple profit equations, we focus on numerical simulations for both brevity and ease of comparison.

pricing with discrimination. What is somewhat surprising is that sequential pricing with discrimination outperforms mixed bundling for large values of θ as well when customers can renege and not trigger price changes. The intuition for this result is that some people who initially passed on A discover the high value from the bundle and decide to return and buy A . For example, a customer may initially pass on a dress, but after observing a pair of shoes that “make the outfit” may decide to purchase both.

Figure 5. Profit with Sequential Pricing with Discrimination – Profit with Mixed Bundling
Under Different Assumptions Regarding Customer Reneging



The third case involving customer reneging is where the seller changes the price of good B based upon the actual purchase of A . This practice is commonplace when $P_B|_{(q_A=1)} < P_B|_{(q_A=0)}$, via coupons that offer a discount on B only with the purchase of A . The results in section 4 suggest that such coupons could be useful for substitutes, when $P^*_B|_{(q_A=1)} < P^*_B|_{(q_A=0)}$. However, for complements the relationship is reversed as the seller wants to offer a lower price to shoppers who do not buy A . Maintaining a relatively high $P_B|_{(q_A=1)}$ could lead consumers to become upset or to make multiple shopping trips if the price difference is greater than their transaction costs. For the moment, we ignore the challenges of maintaining $P_B|_{(q_A=1)} > P_B|_{(q_A=0)}$ and consider the profitability of sequential pricing with discrimination when customers can renege on their A decision but would face adjusted prices for B . In this case, at the second stage consumers would face three prices, P_A , $P_B|_{(q_A=0)}$, and $P_B|_{(q_A=1)}$, and could decide to buy A only, B only, both A and B , or nothing. This decision is computationally identical to that faced by a consumer under mixed bundling. At the optimum, the firm setting sequential prices would set the same prices for A and B alone that a mixed building firm would set. The sequentially

discriminating firm would set $P_B|_{(q_A=1)}$ equal to the difference between the optimal bundle price and P_A . Under our assumption of buyer values being uniformly distributed over a square, a firm engaging in mixed bundling would optimally set the bundle prices below the sum of the individual item prices for all values of θ that we consider and thus it would always be the case that the sequentially discriminating firm would be setting $P_B|_{(q_A=1)} < P_B|_{(q_A=0)}$. Therefore, as reflected in Figure 5, sequential pricing with renegeing where B prices are based upon the ultimate decision to purchase A is a method for implementing mixed bundling over the space considered.

5.2 Rational Buyers

Our analysis in section 4 compares the results of various pricing strategies applied to myopic buyers. How does this assumption affect the results? First, we note that the practices of pure components and mixed bundling are not affected by this assumption. In both cases, buyers are presented with all information simultaneously and therefore neither party is concerned with extracting information to be used later. Clearly, sequential pricing does involve the revelation of information and therefore the optimal prices will be affected by the degree to which the buyer is forward looking. With such buyers the price strategy (in a game theoretic sense) must satisfy both individually rational and incentive compatibility constraints for each buyer type. This constitutes a signaling game in which buyers types may be revealed to some degree and buyers take this into account as they choose their purchase decision.

Reconsidering the simple example presented in the introduction where there are two types of buyers with perfectly correlated values, reveals the issue associated with this type of model. With forward looking buyers, the maximum profit from a pure strategy equilibrium would be $2v+V$. A separating equilibrium cannot be supported the level of profit generated with myopic buyers ($2V+v$) because if the seller charged V for good A and only a type H would buy it, then the seller would always want to charge V for good B when A was purchased and charge v for good B when it was not. This discourages a type H from purchasing good A in order to appear as a type L , and, as a result, the seller will sell no units of good A and only sell good B to type H at a price of V . This encourages the seller to offer good A at a price of v , making a sale to everyone, and then charge V for good B , which only type H buyers will accept. It is worth noting that in this case a seller would prefer to employ bundling to sequential pricing as it reduces the buyer's ability to act strategically.

There are mixed strategy equilibria as well, a formal analysis of which is presented in Appendix C. The appendix illustrates the flavor of a model with fully rational buyers. However, extension of this framework to a general distribution of buyers where values could be sub- or super additive is well beyond the scope of this paper. As mentioned in the introduction, it seems unlikely that human buyers are capable of fully recognizing the impact that each purchase they make may have on the array of goods and prices they subsequently may face. Future research should examine empirically whether or not buyers are myopic. For example in a situation which has some parallels, Bapna, Chang, Goes, and Gupta (2009) point out that extant research does not provide an equilibrium bidding strategy for multiple unit overlapping auction markets; however, they are able to draw conclusions based upon behavioral observation in the field.

6. Conclusions

New technologies will enable sellers to engage in new pricing strategies, and it is important to anticipate how these strategies are likely to affect sellers and customers. Currently, there is a growing trend in retail markets to track individual items. RFID tags or similar ubiquitous technologies can be used to identify which items a buyer intends to purchase at a given price, just as placing an item in an electronic shopping cart does for an e-tailer. Currently, sellers openly use this information to manage inventory and make recommendations regarding other products. However, this information could also be used to adjust prices on items a shopper is likely to purchase.

The purpose of this paper is not about assessing specific technologies for the feasibility of pricing strategy implementation, although commercially available intelligent shelves and long range zonal RFID readers are already in use, and the decreasing costs of tags imply that implementation may already be plausible. This research is about how such technological advances uniquely impact pricing strategies as such capabilities become feasible. *“Marketers recognize that marketing programs intended for different segments—be they products, prices, or advertising messages—must embody different benefits, and if these benefits are well chosen then consumers will themselves choose what they are supposed to choose”* (Moorthy 1984). What are the likely implications of sellers being able to set prices sequentially and discriminate based upon the previous actions of a buyer? As a first step, this paper presents a theoretical model that can be used to answer this question for monopoly markets. The results indicate that the ability to set prices sequentially, absent the ability to discriminate, increases profits relative to a pure components framework where the monopolist sets a price for each good simultaneously when the

goods are weak substitutes. This is due to sequential pricing reducing intra-seller competition between products sold by the same retailer. Further, sequential pricing with conditional pricing, a form of behavior based price discrimination, is more profitable than mixed bundling when the goods are either close substitutes or when the goods are highly positively correlated.

This research is only a first step in studying sequential conditional pricing and it raises several important avenues for future research. How will buyers react to such practices? When Coca-Cola developed a vending machine that adjusted the price with the outside temperature, a negative public backlash thwarted implementation. More generally, Kahneman, et al. (1986) report that people deem it “unfair to exploit shifts in demand by raising prices.” (p. 728). As pointed out by Acquisti (2006, p2.),

“Nobody likes to pay for the same product more than the amount the other person spent. Faced by intrusive information policies and price discrimination strategies, however, consumers can decide to bypass the seller’s tracking attempts through privacy enhancing and anonymizing technologies, or to avoid the seller altogether.”

Of course, people knowingly pay different prices on everything from movie tickets (with discounts for the elderly) to college tuition (with scholarships for students with high test scores). In part, popular acceptance is affected by whether the practice is an industry norm (Heyman and Mellers 2008). While offering the opinion that variable consumer pricing at the individual level will eventually become the norm they offer framing of the practice as a means for firms to make the practice more acceptable to customers. Kimes and Wirtz (2003) found that consumers believed it was fair for a golf course to charge a regular price for early tee times and to offer a 20% discount for later times. Consumers however believed it unfair for a golf course to add a 20% premium for early tee times and to charge a regular price for later times. Thus, some implementations may be more palatable to consumers such as goods that are weak substitutes. For example, when θ is slightly less than 0 we found that sequential pricing without discrimination outperforms mixed bundling and it is easy to imagine a customer who buys one book being offered a discount for a similar although distinct book.

Would consumers stand for being monitored or will their reaction possibly trigger intervention by the Federal Trade Commission (Weiss and Mehrotra 2001)? The very possibility of regulation may cause firms to alter their strategies. For example, Amazon.com varied prices for some electronics based upon shopper’s cookies, but backed off due to social pressure and regulatory threats. Research by Dinev and Hart (2006) provides evidence that while online privacy is a concern, risk perceptions are outweighed by

the benefits of engaging in e-commerce transactions. Therefore, many computer users allow cookies given the improvement they create in the overall internet experience despite the fact that it is difficult to determine if a firm is monitoring cookies and using the information to price discriminate.¹⁹ Many retailers, including Albertson's through its Preferred Shopper Program, offer discounts for people voluntarily identifying themselves to be tracked across visits. Casual observation indicates that people, including one of the authors of this paper, are willing to put up with the privacy invasion for a savings of a few dollars. Future research along the lines of Acquisti and Varian (2005) who consider how sellers might optimally price when buyers can conceal their history is certainly warranted.

How would buyers attempt to strategically shop? Would people make several trips into a store from the parking lot or have friends buy products to save a few cents? The answer is probably not as long as the transaction and contracting costs are greater than the savings, but those few cents could generate a large increase in profit for a retailer serving many customers. After all, at the grocery one rarely finds people selling coupons and looking for partners to take advantage of price cuts via "buy one get one half off" coupons. Would buyers be less likely to visit sellers who engage in sequential conditional pricing? Perhaps they would if the practice is not universally adopted. Ironically, Albertson's used to advertise that they did not engage in a Preferred Shopper Program.

It is important to note that the ability to set sequential prices does not only help the seller, but it is also potentially beneficial to buyers. In the numerical example presented in the introduction, under sequential pricing with discrimination, person L is able to purchase good B , which would not be the case otherwise. It is also easy to imagine situations where the B product is one with which the buyer was unfamiliar initially. The desire to generate profits will lead sellers to make buyers aware of more potentially valuable products. Consumers may be willing to forgo privacy in exchange for more and better recommendations about new products. Ayres (2007) lists many naturally occurring examples of collaborative filtering generating recommended products that the buyer would not have been aware of otherwise, thus increasing a buyer's overall satisfaction. The tactical measures that retailers may employ as a result of this research should be viewed in the context of contemporary initiatives by firms such as Sam's Club to fully automate the retail shopping experience (Weier 2008). Roussos (2006) describes such a ubiquitous technology-enabled retail environment in which a shopper may place products in a smart-cart and check out automatically. They describe a study in which shoppers in such

¹⁹ A 2005 CNN.com article on this topic is available at <http://www.cnn.com/2005/LAW/06/24/ramasastry.website.prices/>.

an environment perceived customized offers as improving the effectiveness of the shopping experience. The technology necessary for sequential pricing could also be used to provide personalized information regarding products such as a product's environmental impact, the percentage of the product made in a certain country, and reviews from similar users all of which shoppers may find sufficiently valuable to forge privacy concerns.

The issue of how consumers would respond to the lack of privacy required for sequential pricing is important, but anything more than speculation would require a full scale behavioral study and is beyond the scope of this paper. As we describe in section 5 in our discussion of renegeing, there are implementation mechanisms that plausibly may be acceptable to consumers despite the potential for sequential pricing to cause perceptions of unfairness or to negatively impact trustworthiness. Offering a coupon for a discount on product B conditional on the purchase of product A is a common practice in retail. Finding a mechanism to raise the price for product B conditional on the purchase of product A is a more difficult problem. This may require a shopping environment in which there are no posted prices, but one where each customer receives a targeted price. Given recent developments with smart carts (Roussos 2006) and with smartphone applications (Clifford 2010b) such an environment may be a reality in the near future. Alternatively, there may be creative solutions available to sellers such as Hinz et al. (2011) who find that using a revealed adaptive threshold price to discriminate can raise profits by 20% without lowering customer satisfaction. All of these possible solutions however must be viewed in the light of customer tradeoffs between privacy and savings. There is evidence that despite strong opinions on privacy customers are not vigilant in monitoring their own behavior reflective of these concerns (Berendt, Günther, and Spiekermann 2005). Finally, Pramataris and Theotokis (2009) provide evidence that the effort expectancy and performance expectancy of Venkatesh, Morris, Davis, and Davis (2003) may negatively and positively, respectively, influence attitude toward RFID-enabled service. The type of recommender mechanisms we propose with sequential pricing require little effort on the part of the customer and are thus likely to be viewed favorably (Haubl and Murray 2006). Customer evaluation of a system that enables sequential pricing is also likely to be a consequence of their perception of the cost savings, and so it is important to make these potential savings salient to them.

Ultimately, the degree to which technology enabled pricing strategies become prevalent is a function of the potential profitability of the practices, which would reflect the degree to which customers are accepting of the practices and the cost of developing and installing the infrastructure necessary to implement such strategies. Further analytical and behavioral studies are needed to provide guidance to

industry as firms navigate this new world of the technology-enabled omniscient retailer, so laden with possibility. The theoretical foundation provided by the current research may help with the initial steps.

References

- Acquisti, A. 2006. Ubiquitous computing, customer tracking, and price discrimination. G. Roussos, ed. *Ubiquitous and Pervasive Commerce: New Frontiers for Electronic Business*. Springer-Verlag, London, UK, 115-132.
- Acquisti, A., H. R. Varian. 2005. Conditioning price on purchase history. *Marketing Sci.* **24**(3) 367-381.
- Adams, W. J., J. Yellen. 1976. Commodity bundling and the burden of monopoly. *Quart. J. Econom.* **40** 475-498.
- ADT Reference Library. An independent report on RFID tracking. Accessed October 29, 2009, http://www.adt.com/medium_large_bus./reference_library/?wgc=Item-Level_RFID_Tracking.
- Arora, N. X., A. Dreze, J. D. Ghose, R. Hess, B. Iyengar, B. Jing, Y. Joshi, V. Kumar, N. Lurie, S. Neslin, S. Sajeesh, M. Su, N. Syam, J. Thomas, Z. J. Zhang. 2008. Putting one-to-one Marketing to Work: Personalization, Customization and Choice. *Marketing Letters* **19**(3-4) 305-321.
- Aviv, Y., A. Pazgal. 2008. Optimal Pricing of Seasonal Products in the Presence of Forward-Looking Consumers. *Manufac. Service Oper. Management* **10**(3) 339-359.
- Aloysius, J., C. Deck, A. Farmer. 2010. A Comparison of Bundling and Sequential Pricing in Competitive Markets: Experiential Evidence" *International Journal of the Economics of Business*, forthcoming
- Ansari, A., S. Essegai, R. Kohli. 2000. Internet recommendation systems. *J. Marketing Res.* **37**(3) 363-375.
- Ayres, I. 2007. *Super Crunchers: Why thinking-by-numbers is the new way to be smart*. Bantam Dell.
- Bandyopadhyay, S., J. M. Barron, A. R. Chaturvadi. 2005. Competition among sellers in online exchanges. *Inform. Systems Res.* **16**(1) 47-60.
- Bapna, R., S-J. Chang, P. Goes, A. Gupta. 2009. Overlapping online auctions: Empirical characterization of bidder strategies and auction prices. *MIS Quart.* **33**(4) 763-783.
- Bapna, R., P. Goes, A. Gupta. 2003a. Replicating online Yankee auctions to analyze auctioneers' and bidders' strategies. *Inform. Systems Res.* **14**(3) 244-268.
- Bapna, R., P. Goes, A. Gupta. 2003b. Analysis and design of business-to-consumer online auctions. *Management Sci.* **49**(1) 85-101.
- Basenko, D., J-P. Dube, S. Gupta. 2003. Competitive price discrimination strategies in a vertical channel using aggregate retail data. *Management Sci.* **49**(9) 1121-1138.
- Berendt, B., O. Günther, S. Spiekermann. 2005. Privacy in E-Commerce: Stated preferences vs. actual behavior. *Communications of the ACM*, **48**(4) 101-106.
- Bergen, M. E., R. J. Kauffman, D. Lee. 2005. Beyond the hype of frictionless markets: Evidence of heterogeneity in price rigidity on the internet. *J. Management Inform. Systems* **22**(2) 57-89.

- Bichler, M., P. Shabalin, A. Pikovsky. 2009. A computational analysis of linear price iterative combinatorial auction formats. *Inform. Systems Res.* **20**(1) 33-59.
- Bodapati, A. V. 2008. Recommendation systems with purchase data. *J. Marketing Res.* **45**(1) 77-93.
- Chen, Y. 1997. Equilibrium product bundling. *J. Bus.* **70**(1) 85-103.
- Chen, Y., C. Narasimhan, Z. J. Zhang. 2001. Individual marketing with imperfect targetability. *Marketing Sci.* **20**(1) 23-41.
- Chen, Y., G. Iyer. 2002. Customer addressability and customized pricing. *Marketing Sci.* **21**(2) 197-208.
- Choudhary, V., A. Ghose, T. Mukhopadhyay, U. Rajan. 2005. Personalized pricing and quality differentiation. *Management Sci.* **51** 1121-1130.
- Clifford, S. 2010a. Web coupons know lot about you, and they tell. New York Times (April 17).
- Clifford, S. 2010b. Aisle by aisle, an app that pushes bargains. New York Times (August 17).
- Cope, E. 2006. Bayesian strategies for dynamic pricing in e-commerce. *Naval Res. Logistics* **54** 265-281.
- Cromhout, D., B. C. Hardgrave, D. J. Armstrong. 2008. RFID item-level tagging for apparel/footwear: A feasibility study. Working paper, Information Technology Research Institute, Walton College of Business, University of Arkansas.
- De Bruyn, A., J. C. Liechty, E. K. R. E. Huizingh, G. L. Lilien. 2008. Offering online recommendations with minimum customer input through conjoint-based decision aids. *Marketing Sci.* **27**(3) 443-460.
- Dhebar, A. 1994. Durable goods monopolists, rational consumers, and improving products. *Marketing Sci.* **13**(1) 100-120.
- DIGIA. 2010. Digia implements price label solution for Tradeka based on electronic paper. Downloaded from <http://www.pressreleasepoint.com/digia-implements-price-label-solution-tradeka-based-electronic-paper> on Feb 5 2010.
- Dinev, T., P. Hart. 2006. An extended privacy calculus model for e-commerce transactions. *Inform. Systems Res.* **17**(1) 61-80.
- Draganska, M., D. C. Jain. 2006. Consumer preferences and product-line pricing strategies: An empirical analysis. *Marketing Sci.* **25**(2) 164-174.
- Ellison, G. 2006. Bounded Rationality in Industrial Organization. R. Burnell, W. Newey, T. Persson, ed. *Advances in Economics and Econometrics: Theory and Applications*. Ninth World Congress, Cambridge University Press, New York, NY.
- Epapercentral. 2009. IBM and the Future of E-Paper in Retail Downloaded from <http://www.epapercentral.com/ibm-and-the-future-of-epape-in-retail.htm> on Feb 5 2010.

- Foubert, B., E. Gijsbrechts. 2007. Shopper response to bundle promotions for packaged goods. *J. Marketing Res.* **44**(4) 647-662.
- Fudenberg, D., J. Tirole. 2000. Customer poaching and brand switching. *RAND J. Economics* **31** 238-258.
- Fudenberg, D., J. M. Villas-Boas. 2006. Behavior based price discrimination and customer recognition. T. J. Hendershott, ed. *Handbook on Economics and Information Systems.* (Elsevier, Oxford) 377-436.
- Gabaix, X., D. Laibson. 2006. Shrouded attributes, consumer myopia, and information suppression in competitive markets. *Quart. J. Economics* **121**(2) 505-540.
- Ghose, A., A. Sundararajan. 2006. Evaluating pricing strategy using e-commerce data: Evidence and estimation challenges. *Statistical Sci.* **21**(2) 131-142.
- Hanson, W. A., R. K. Martin. 1990. Optimal bundle pricing. *Management Sci.* **36**(2) 155-174.
- Happich, J. 2010. Custom electronic paper display drivers target electronic shelf labels. Downloaded from http://www.electronics-eetimes.com/en/custom-electronic-paper-display-drivers-target-electronic-shelf-labels.html?cmp_id=7&news_id=222900962# on Feb 5 2010.
- Häubl, G., K. B. Murray. 2006. Double agents: Assessing the role of electronic product recommendation systems. *MIT Sloan Management Review* **47**(3) 8-12.
- Hess, J. D., E. Gerstner. 1987. Loss leader pricing and rain check policy. *Marketing Sci.* **6**(4) 358-374.
- Heyman, J., B. A. Mellers. 2008. Perceptions of fair pricing. In Haugtvedt, C., Kardes, F., & Herr, P. (Eds.) *Handbook of Consumer Psychology.* Psychology Press, New York, NY.
- Hinz, O., I-H Hann, M. Spann. 2011. Price discrimination in e-commerce? An examination of dynamic pricing in name-your-own price markets. *MIS Quart.* **35**(1) 81-98.
- Huang, Z., D. D. Zeng, C. Chen. 2007. Analyzing consumer-product graphs: Empirical findings and applications in recommender systems. *Management Sci.* **53**(7) 1146-1164.
- Jiang, Y., Shang, J., C. F. Kemerer, Y. Liu, Y. 2011. Optimizing e-tailer profits and customer savings: an online dynamic bundle pricing model. *Marketing Science*, forthcoming.
- Kahneman, D., J. Knetsch, R. Thaler. 1986. Fairness as a constraint on profit seeking: entitlements in the market. *Amer. Econom. Rev.* **76**(4) 728-741.
- Kamakura, W. A., B. S. Kossar, M. Wedel. 2004. Identifying innovators for the cross-selling of new products. *Management Sci.* **50**(8) 1120-1133.
- Kettler, P. C. 2007. The pyramid distribution. Working Paper, University of Oslo.
- Kimes, S. E., J. Wirtz. 2003. *Journal of Revenue and Pricing Management* **1**(4) 332-344. Kourouthanassis, P., G. Roussos. 2006. The design of pervasive retail experiences. G. Roussos, ed. *Ubiquitous and Pervasive Commerce: New Frontiers for Electronic Business.* Springer-Verlag, London, UK, 133-153.

- Kannan, P. K., P. K. Kopalle. 2001. Dynamic pricing on the internet: Importance and implications for consumer behavior. *International Journal of Electronic Commerce*, 5 (Spring), 63-84.
- Leszczyc, P., G. Haubl. 2010. To bundle or not to bundle: Determinants of the profitability of multi-item auctions. *J. Marketing*. Forthcoming.
- Maglaras, C., J. Meissner. 2006. Dynamic pricing strategies for multi-product revenue management problems. *Manufac. Service Operations Management* **8**(2) 136-148.
- March, J. G. 1994. *A primer on decision making*. Free Press, New York, NY.
- Martin, A. 2010. MasterCard set to open an online shopping mall. *New York Times* (April 8)
- McAfee, R. P., J. McMillan, M. D. Whinston. 1989. Multiproduct monopoly, commodity bundling, and correlation of values. *Quart. J. Econom.* **103** 371–383.
- McAfee, R. P., D. Vincent. 1995. Sequentially optimal auctions. *Games and Economic Behavior* **18** 246-276.
- Montgomery, A. L., S. B. Li, K. Srinivasan, J. C. Liechty. 2004. Modeling online browsing and path analysis using clickstream data. *Marketing Sci.* **23**(4) 579-595.
- Moon, S., G. J. Russell. 2008. Predicting product purchase from inferred customer similarity: An autologistic model approach. *Management Sci.* **54**(1) 71-82.
- Moorthy, K. S. 1984. Market segmentation, self selection, and product line design. *Marketing Sci.* **3**(4) 288-307.
- Moorthy, K. S., I. Png. 1992. Market segmentation, cannibalization and the timing of product introductions. *Management Sci.* **38**(3) 345-359.
- Mulhern, F. J., R. P. Leone. 1991. Implicit price bundling of retail products: A multiproduct approach to maximizing store profitability. *J. Marketing* **55**(4) 63-76.
- Natter, M., T. Reutterer, A. Mild, A. Taudes. 2007. An assortment wide decision-support system for dynamic pricing and promotion planning in DIY retailing. *Marketing Sci.* **226**(4) 576-583.
- Netessine, S., S. Savin, W. Xiao. 2006. Revenue management through dynamic cross selling in e-commerce retailing. *Oper. Res.* **54**(5) 893-913.
- Payne, J.W. & Bettman, J.R. 2001. Preferential choice and adaptive strategy use. In G. Gigerenzer & R. Selten (Eds.) *Bounded Rationality: The Adaptive Toolbox*. MIT Press, Cambridge, MA, 123-145.
- Pigou, A. C. 2006. *The Economics of Welfare: Vol. 1*. Cosimo, New York: NY.
- Pramatari, K., Theotokis, A. 2009. Consumer acceptance of RFID-enabled services: A model of multiple attitudes, perceived system characteristics and individual traits. *European Journal of Information Systems* **18** 541–552

- Rossi, P. E., R. E. McCulloch, G. M. Allenby. 1996. The value of purchase history data in target marketing. *Marketing Sci.* **15**(4) 321-340.
- Rust, R. T., T. S. Chung. 2006. Marketing models of service and relationships. *Marketing Sci.* **25**(6) 560-580.
- Schmalensee, R. 1984. Pricing of product bundles. *J. Bus.* **57**(1) 211-230.
- Shaffer, G., Z. J. Zhang. 2002. Competitive one-to-one promotions. *Management Sci.* **48**(9) 1143-1160.
- Shaffer, G., Z. J. Zhang. 2000. Pay to switch or pay to stay: Preference based price discrimination in markets with switching costs. *J. Econom. Management Strategy* **9**(3) 397-424.
- Shaffer, G., Z. J. Zhang. 1995. Competitive coupon targeting. *Marketing Sci.* **14**(4) 395-416.
- Shell, E. R. 2009. Cheap: The High Cost of Discount Prices. Penguin: New York, NY.
- Subramaniam, R., R. Venkatesh. 2009. Optimal bundling strategies in multiobject auctions of complements or substitutes. *Marketing Sci.* **28**(2) 264-273.
- Ulph, D., N. Vulkan. 2000. *Electronic Commerce and Competitive First-Degree Price Discriminators*. Working Paper, ESCR Centre for Economic Learning and Social Evolution, University College London.
- US Census Bureau (2008. Quart. retail e-commerce sales 4th quarter 2007, downloaded from <http://www.census.gov/mrts/www/data/html/07Q4.html>, on 9/11/08.
- Varian, H. R. 1980. A model of sales. *Amer. Economic Rev.* **70**(4) 651-659.
- Viswanath, V., M. G. Morris, G. B. Davis, F. D. Davis. 2003. User acceptance of information technology: Toward a unified view, *MIS Quarterly* **27** (3) 425-78.
- Venkatesh, R., W. Kamakura. 2003. Optimal bundling and pricing under a monopoly: Contrasting complements and substitutes from independently valued products. *J. Bus.* **76**(2) 211-231.
- Viswanathan, S., J. Kuruzovich, S. Gosain, R. Agarwal. 2007. Online infomediaries and price discrimination: Evidence from the automotive retailing sector. *J. Marketing* **71**(3) 89-107.
- Vulkan, N. 2003. *The economics of e-commerce: A strategic guide to understanding and designing the online marketplace*. Princeton University Press, Princeton, NJ.
- Weier, M. H. 2008. Walmart gets tough on RFID with Sam's Club suppliers. *Inform. Week* **1169** (January 19) 26.
- Weiss, R. M., A. K. Mehrotra. 2001. Online dynamic pricing: Efficiency, equity and the future of e-commerce. *Virginia J. Law and Tech.* **6**(11) at <http://www.vjolt.net>.
- Zhang, J. 2009. Competitive targeted pricing: perspectives from theoretical Research. V. E. Rao, ed. *Handbook of Pricing Research in Marketing*. Edward Elgar Publishing, Northampton, MA, 302-318.
- Zhang, J., L. Krishnamurthi. 2004. Customizing promotions in online stores. *Marketing Sci.* **23**(4) 561-578.

APPENDIX A: SEQUENTIAL PRICING WITHOUT PRICE DISCRIMINATION

Stage 2: Choose P_B .

Recall that without price discrimination, P_A has been chosen, but the purchase of good A is not observed. All that is known is the probability that A was chosen given the distribution of preferences, and this probability will affect the optimal choice of P_B if the goods are related in any way. The monopolist will maximize expected profit as follows:

$$E\Pi(P_B|P_A) = (P_B - C_B) \int_{P_B}^{\infty} \int_0^{P_A} f(V_A)f(V_B)dV_A dV_B + (P_B - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f(V_A)f(V_B)dV_B dV_A$$

$$\begin{aligned} \frac{\partial E\Pi}{\partial P_B} &= (P_B - C_B)(-f(P_B)) \int_0^{P_A} f(V_A)dV_A + \int_{P_B}^{\infty} \int_0^{P_A} f(V_A)f(V_B)dV_A dV_B \\ &\quad + (P_B - C_B) \left(-\frac{f(P_B)}{1 + \theta} \right) \int_{P_A}^{\infty} f(V_A)dV_A + \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f(V_A)f(V_B)dV_B dV_A \end{aligned}$$

Setting this equal to 0, simplifying and rearranging we arrive as the following FOC.

$$\begin{aligned} (P_B - C_B)(-f(P_B)) \left[F(P_A) + \frac{1}{1 + \theta} - \frac{F(P_A)}{1 + \theta} \right] + 1 - F(P_B)F(P_A) \\ - \int_{P_A}^{\infty} f(V_A)F\left(\frac{P_B - \theta V_A}{1 + \theta}\right) dV_A = 0 \end{aligned} \quad (A1)$$

Where $F(P)$ represents the cumulative distribution. While solving explicitly for P_B is not possible without specifying a distribution, equation (A1) can be solved implicitly for $P_B^*(P_A, \theta)$

Stage 1: Solve for P_A

Knowing how the price of B will be chosen conditional on the price of A, the monopolist will now choose P_A to maximize expected profits from both stages of the game.

$$\begin{aligned} E\Pi(P_A) &= (P_A - C_A) \int_{P_A}^{\infty} \int_0^{\frac{P_B^* - \theta V_A}{1 + \theta}} f(V_A)f(V_B)dV_B dV_A + (P_B^* - C_B) \int_{P_B}^{\infty} \int_0^{P_A} f(V_A)f(V_B)dV_A dV_B \\ &\quad + (P_A - C_A + P_B^* - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f(V_A)f(V_B)dV_B dV_A \end{aligned}$$

Where P_B^* solves (A1). Integrating the expected profit and simplifying yields a more manageable profit expression as follows:

$$E\Pi(P_A) = (P_B^* - C_B) \left[1 - F(P_A)F(P_B^*) - \int_{P_A}^{\infty} f(V_A)F\left(\frac{P_B^* - \theta V_A}{1 + \theta}\right) dV_A \right] + (P_A - C_A)[1 - F(P_A)]$$

Substituting in from (A1) for $\int_{P_A}^{\infty} f(V_A)F\left(\frac{P_B^* - \theta V_A}{1 + \theta}\right) dV_A$ yields

$$E\Pi(P_A) = (P_B^* - C_B)^2 f(P_B^*) \left[F(P_A) + \frac{1}{1 + \theta} - \frac{F(P_A)}{1 + \theta} \right] + (P_A - C_A)(1 - F(P_A))$$

where P_B^* solves (A1). Performing the maximization provides the first order condition found in (A2).

$$\begin{aligned} \frac{dE\Pi(P_A)}{dP_A} &= P_B^{*2} f(P_B^*) \left[(f(P_A)) \left(1 - \frac{1}{1 + \theta} \right) \right] + 1 - F(P_A) - P_A f(P_A) \\ &+ \left[F(P_A) + \frac{1}{1 + \theta} - \frac{F(P_A)}{1 + \theta} \right] \left[P_B^{*2} f'(P_B) + 2P_B f(P_B) \frac{dP_B^*}{dP_A} \right] = 0 \end{aligned} \quad (A2)$$

Optimum:

Equations (A1) and (A2) fully characterize the optimal choice of prices for any general continuous distributions of values.

Comparative Statics:

From (1), we perform a comparative statics exercise to find dP_B / P_A . For simplicity, let $C_B = 0$.

Totally differentiate to get:

$$\begin{aligned} -[P_B f'(P_B) + f(P_B)] &\left[F(P_A) + \frac{1}{1 + \theta} - \frac{F(P_A)}{1 + \theta} \right] dP_B - F(P_A) f(P_B) dP_B \\ &- \left(\frac{1}{1 + \theta} \int_{P_A}^{\infty} f(V_A) f\left(\frac{P_B - \theta V_A}{1 + \theta}\right) dV_A dP_B \right) - f(P_A) \left(1 - \frac{1}{1 + \theta} \right) (P_B f(P_B)) dP_A \\ &- F(P_B) f(P_A) dP_A + f(P_A) F\left(\frac{P_B - \theta P_A}{1 + \theta}\right) dP_A = 0 \end{aligned}$$

Rearranging the total differential, we find

$$\frac{dP_B}{dP_A} = \frac{f(P_B)P_B f(P_A) \left(1 - \frac{1}{1+\theta}\right) + F(P_B)f(P_A) - f(P_A)F\left(\frac{P_B - \theta P_A}{1+\theta}\right)}{-\left[F(P_A) + \frac{1}{1+\theta} \frac{F(P_A)}{1+\theta}\right] [P_B f'(P_B)] - F(P_A)f(P_B) - \frac{1}{1+\theta} \int_{P_A}^{\infty} f(V_A) f\left(\frac{P_B - \theta V_A}{1+\theta}\right) dV_A} \quad (A3)$$

We know from the second order condition that the denominator is < 0 . Therefore, the sign of $\frac{dP_B}{dP_A}$ is opposite sign of numerator, or $\text{sgn}\left(\frac{dP_B}{dP_A}\right) = -\text{sgn}(\text{numerator})$ of (A3). When signing the numerator we need to separately consider the cases in which θ is both positive and negative.

Case 1: $\theta > 0$:

This implies that $1 - \frac{1}{1+\theta} > 0$. Thus, the first term in the numerator of (A3) is > 0 .

One can combine last two terms to get $f(P_A) \left[F(P_B) - F\left(\frac{P_B - \theta P_A}{1+\theta}\right) \right]$. When $\theta > 0$ we know $\frac{P_B - \theta P_A}{1+\theta} < P_B$ and, therefore, $F\left(\frac{P_B - \theta P_A}{1+\theta}\right) < F(P_B)$. It follows then that this term is > 0 .

Therefore, the sign of the numerator is > 0 , and $\frac{dP_B}{dP_A} < 0$ when $\theta > 0$.

Case 2: $-1 < \theta < 0$:

First note that θ will never be less than -1 because conceptually, a value below minus 1 would imply that the bundle would have an overall negative value. When θ is negative, $1 - \frac{1}{1+\theta} < 0$ making the first term in the numerator of (3) positive. The second and third terms, which have been rewritten as $f(P_A) \left[F(P_B) - F\left(\frac{P_B - \theta P_A}{1+\theta}\right) \right]$, is negative because when $\theta < 0$, $\frac{P_B - \theta P_A}{1+\theta} > P_B$ and thus $F\left(\frac{P_B - \theta P_A}{1+\theta}\right) > F(P_B)$. Therefore, $\frac{dP_B}{dP_A} > 0$ when $\theta < 0$.

This leads to Proposition 1.

Proposition 1: When a monopolist engages in sequential pricing without discrimination and the goods are substitutes (complements) the price of good B is decreasing (increasing) in the price of good A. That is $\theta < 0$, $\frac{dP_B}{dP_A} > 0$ and when $\theta > 0$, $\frac{dP_B}{dP_A} < 0$.

Now consider the impact of θ on the prices of goods A and B. Let us rewrite the first order conditions (A1) and (A2) in general notation. Rewrite (A1) as the following implicit general function

$$f(P_B, P_A, \theta) = 0 \quad (A1')$$

where P_B is implicitly a function of P_A and θ .

To find $\frac{dP_B}{d\theta}$, we totally differentiate (A1') to get:

$$\frac{dP_B}{d\theta} = \frac{-\left(\frac{df}{dP_A} \frac{dP_A}{d\theta} + \frac{df}{d\theta}\right)}{\frac{df}{dP_B}}$$

From second order conditions we know that the denominator is negative, and thus

$$\text{Sgn}\left(\frac{dP_B}{d\theta}\right) = \text{sgn}\left(\frac{df}{dP_A} \frac{dP_A}{d\theta} + \frac{df}{d\theta}\right).$$

Solving for each of these derivatives from the actual function f in equation (A1), we find the following:

$$\frac{df}{d\theta} = P_B f(P_B) [1 - F(P_B)] \frac{1}{(1+\theta)^2} + \int_{P_A}^{\infty} f(V_A) f\left(\frac{P_B - \theta V_A}{1+\theta}\right) \frac{P_B + V_A}{(1+\theta)^2} dV_A dP_B > 0$$

and

$$\frac{df}{dP_A} = -P_B f(P_B) \left(1 - \frac{1}{1+\theta}\right) f(P_A) + f(P_A) \left[F\left(\frac{P_B - \theta P_A}{1+\theta}\right) - F(P_B)\right]$$

So, $\frac{df}{dP_A} > 0$ when $\theta < 0$, and is < 0 when $\theta > 0$. However, we need to find $\frac{dP_A}{d\theta}$ which proves much more difficult. To do so, rewrite (A2) implicitly in general form as

$$g(P_A, P_B(P_A, \theta), \theta) = 0 \quad (\text{A2}')$$

Totally differentiating (2') and solving for $\frac{dP_A}{d\theta}$, we get:

$$\frac{dP_A}{d\theta} = \frac{-\left(\frac{dg}{dP_B} \frac{\delta P_B}{\delta \theta} + \frac{dg}{d\theta}\right)}{\frac{dg}{dP_A} + \frac{dg}{dP_B} \frac{dP_B}{dP_A}}$$

Where the denominator is negative by second order conditions, and thus,

$$\text{Sgn}\left(\frac{dP_A}{d\theta}\right) = \text{sgn}\left(\frac{dg}{dP_B} \frac{dP_B}{d\theta} + \frac{dg}{d\theta}\right)$$

Let us consider the sign of each of these pieces in turn. First note that the direct effect of θ on P_B , $\frac{dP_B}{d\theta} > 0$. This is $\frac{df}{d\theta}$ from above equation (2'). The other pieces of $\text{sgn}\left(\frac{dg}{dP_B} \frac{dP_B}{d\theta} + \frac{dg}{d\theta}\right)$ prove more difficult to sign.

Recall from (A2) that

$$g(P_A, P_B(P_A, \theta), \theta) = P_B^2 f(P_B^*) \left[(f(P_A)) \left(1 - \frac{1}{1+\theta}\right) \right] + 1 - F(P_A) - P_A f(P_A) \\ + \left[F(P_A) + \frac{1}{1+\theta} - \frac{F(P_A)}{1+\theta} \right] \left[P_B^2 f'(P_B) + 2P_B f(P_B) \frac{dP_B^*}{dP_A} \right]$$

The expression for $\frac{dg}{d\theta}$ can be found directly from differentiating (A2) to find:

$$\frac{dg}{d\theta} = P_B^2 f(P_B) f(P_A) \frac{1}{(1+\theta)^2} + [P_B^2 f'(P_B) + 2P_B f(P_B) \frac{dP_B}{dP_A}] [1 - F(P_A)] \left[-\frac{1}{(1+\theta)^2} \right]$$

$$+ \left[F(P_A) + \frac{1}{1+\theta} - \frac{F(P_A)}{1+\theta} \right] 2P_B f(P_B) \frac{d^2 P_B}{dP_A d\theta}$$

and,

$$\frac{dg}{dP_B} = (f(P_A)) \left(1 - \frac{1}{1+\theta} \right) [2P_B f(P_B) + f'(P_B) P_B^2] + \left[F(P_A) + \frac{1}{1+\theta} - \frac{F(P_A)}{1+\theta} \right] [2P_B f'(P_B) + f''(P_B) P_B^2 + 2 \frac{dP_B}{dP_A} (f(P_B) + P_B f'(P_B))]$$

Since f can be of any shape, the signs of f' and f'' are indeterminate. Moreover, we would need to sign the cross partial $\frac{d^2 P_B}{dP_A d\theta}$, which involves differentiating (3) with respect to θ . As such, arriving at a sign for $\frac{dP_A}{d\theta}$ is intractable, as is determining the sign for $\frac{dP_B}{d\theta}$ which depends upon the sign of $\frac{dP_A}{d\theta}$.

However, while a solution for the impact of changes in θ on prices proves difficult, the impact on profit can be found. Using the chain rule, we know that

$$\frac{d\Pi^*}{d\theta} = \frac{d\Pi}{d\theta} + \frac{d\Pi}{dP_A} \frac{dP_A^*}{d\theta} + \frac{d\Pi}{dP_B} \frac{dP_B}{d\theta} + \frac{d\Pi}{dP_B} \frac{dP_B}{dP_A} \frac{dP_A}{d\theta}$$

By the envelope theorem, the last 3 terms = 0 and recall

$$\Pi^*(P_A^*) = (P_B^* - C_B) \left[1 - F(P_A) F(P_B^*) - \int_{P_A}^{\infty} f(V_A) F\left(\frac{P_B^* - \theta V_A}{1+\theta}\right) dV_A \right] + (P_A^* - C_A) [1 - F(P_A^*)]$$

So,

$$\frac{d\Pi^*(P_A^*)}{d\theta} = - \int_{P_A}^{\infty} f(V_A) F\left(\frac{P_B^* - \theta V_A}{1+\theta}\right) dV_A \left(f\left(\frac{P_B^* - \theta V_A}{1+\theta}\right) \right) \left(- \frac{(1+\theta) - \theta}{(1+\theta)^2} \right) > 0$$

This leads to Proposition 2.

Proposition 2: When a monopolist engages in sequential pricing without discrimination, expected profit is increasing in θ . In other words, as intuition would suggest, as goods become more complementary, sequential pricing results in greater expected profits.

APPENDIX B: SEQUENTIAL PRICING WITH DISCRIMINATION

Stage 2: Choose P_B .

Recall that with price discrimination, P_A has been chosen and the purchase of good A is observed. The monopolist will maximize expected profit differently depending upon whether A was purchased. Consider first the case in which A was not purchased.

Case 1: $q=0$; A was not purchased.

$$E\Pi(P_B|P_A)_{q=0} = (P_B - C_B) \int_{P_B}^{\infty} f(V_B) dV_B = (P_B - C_B)(1 - F(P_B))$$

Taking the first order condition we know:

$$\frac{dE\Pi(P_B)}{dP_B} = (P_B - C_B)(-f(P_B)) + 1 - F(P_B) = 0 \quad (B1a)$$

Note that P_B will be independent of P_A and θ when A is not purchased.

Case 2: $q=1$; A was purchased

$$E\Pi(P_B|P_A)_{q=1} = (P_B - C_B) \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f(V_A|V_A > P_A) f(V_B) dV_B dV_A$$

Which yields the first order condition below:

$$\begin{aligned} \frac{dE\Pi}{dP_B} = (P_B - C_B) \int_{P_A}^{\infty} f(V_A|V_A > P_A) f(V_B) \left(-\frac{1}{1 + \theta} \right) dV_A \\ + \int_{P_A}^{\infty} \int_{\frac{P_B - \theta V_A}{1 + \theta}}^{\infty} f(V_B) f(V_A|V_A > P_A) dV_B dV_A = 0 \end{aligned} \quad (B1b)$$

Simplifying and rearranging yields

$$(P_B - C_B) \left[f(P_B) \left(-\frac{1}{1 + \theta} \right) (1 - F(P_A|P_A > P_A)) \right] + \int_{P_A}^{\infty} f(V_A|V_A > P_A) \left[1 - F\left(\frac{P_B - \theta V_A}{1 + \theta}\right) \right] dV_A = 0$$

Note that $F(P_A|P_A > P_A) = 0$, and rewrite (B1b) as

$$-(P_B - C_B) \frac{f(P_B)}{1 + \theta} + \int_{P_A}^{\infty} f(V_A|V_A > P_A) \left[1 - F\left(\frac{P_B - \theta V_A}{1 + \theta}\right) \right] dV_A = 0 \quad (B1b')$$

Stage 1: Solve for P_A

Knowing how the price of B will be chosen conditional on the price of A, the monopolist will now choose P_A to maximize expected profits from both stages of the game.

$$\begin{aligned} E\Pi(P_A|P_{B_0}^*, P_{B_1}^*) &= \text{Prob}(q = 0)E\Pi(P_A|q = 0) + \text{Prob}(q = 1)E\Pi(P_A|q = 1) \\ &= F(P_A)P_{B_0}^* (1 - F(P_{B_0}^*)) + [1 - F(P_A)]P_{B_1}^* \int_{P_A}^{\infty} \int_{\frac{P_{B_1}^* - \theta V_A}{1 + \theta}}^{\infty} f(V_B)f(V_A|V_A > P_A)dV_B dV_A \end{aligned}$$

Where $P_{B_0}^*$ solves (1a) and $P_{B_1}^*$ solves (B1b)

Simplify the expected profit expression by substituting first order condition (B1b) in for the last term:

$$E\Pi(P_A|P_B^*) = F(P_A)P_{B_0}^* (1 - F(P_{B_0}^*)) + (1 - F(P_A))P_{B_1}^* \left[\frac{P_{B_1}^* f(P_{B_1}^*)}{1 + \theta} \right]$$

Differentiate to find the first order condition in (2).

$$\begin{aligned} \frac{dE\pi}{dP_A} &= P_{B_0}^* (1 - F(P_{B_0}^*)) f(P_A) + F(P_A) [P_{B_0}^* (-f(P_{B_0}^*)) + (1 - F(P_{B_0}^*))] \frac{dP_{B_0}^*}{dP_A} \\ &\quad + \frac{P_{B_1}^{*2} f(P_{B_1}^*)}{1 + \theta} (-f(P_A)) + \left(\frac{1 - F(P_A)}{1 + \theta} \right) (P_{B_1}^{*2} f'(P_{B_1}^*) + 2P_{B_1}^* f(P_{B_1}^*)) \frac{dP_{B_1}^*}{dP_A} = 0 \end{aligned} \quad (B2)$$

Since $\frac{dP_{B_0}^*}{dP_A} = 0$, the expressions simplifies to

$$\begin{aligned} P_{B_0}^* (1 - F(P_{B_0}^*)) f(P_A) + \frac{P_{B_1}^{*2} f(P_{B_1}^*)}{1 + \theta} (-f(P_A)) + \\ \left(\frac{1 - F(P_A)}{1 + \theta} \right) (P_{B_1}^{*2} f'(P_{B_1}^*) + 2P_{B_1}^* f(P_{B_1}^*)) \frac{dP_{B_1}^*}{dP_A} = 0 \end{aligned}$$

This describes the first order condition for P_A where $P_{B_0}^*$ solves (B1a) and $P_{B_1}^*$ solves (B1b').

Optimum:

Equations (B1a), (B1b') and (B2) fully characterize the optimal choice of prices for any general continuous distributions of values.

Comparative Statics:

Recall that in (B1a), P_B is independent of P_A . From (B1b'), we perform a comparative statics exercise to find dP_B / P_A . For simplicity, let $C_B = 0$.

$$\left[-\frac{P_B}{1+\theta} f'(P_B) - \frac{f(P_B)}{1+\theta} - \frac{1}{1+\theta} \int_{P_A}^{\infty} f(V_A|V_A > P_A) f\left(\frac{P_B^* - \theta V_A}{1+\theta}\right) dV_A \right] dP_B$$

$$-f(V_A|V_A > P_A) \left(1 - F\left(\frac{P_B^* - \theta P_A}{1+\theta}\right) \right) dP_A = 0$$

Rearranging and simplifying we find:

$$\frac{dP_B}{dP_A} = \frac{f(V_A|V_A > P_A) \left(1 - F\left(\frac{P_B^* - \theta P_A}{1+\theta}\right) \right)}{\left[-\frac{P_B}{1+\theta} f'(P_B) - \frac{f(P_B)}{1+\theta} - \frac{1}{1+\theta} \int_{P_A}^{\infty} f(V_A|V_A > P_A) f\left(\frac{P_B^* - \theta V_A}{1+\theta}\right) dV_A \right]} < 0$$

because the second order condition makes denominator negative and the numerator is positive. As the price of A rises, the price of B will be lower conditional on having purchased A regardless of whether the goods are substitutes or complements.

This leads to proposition 3.

Proposition 3: When a monopolist engages in sequential pricing with discrimination, the optimal price of good B for customers who buy A is decreasing in the price of good A regardless of the degree of complementarity. That is when A was purchased, $\frac{dP_B|_{(q=1)}}{dP_A} < 0$ regardless of the sign of θ .

Now consider the impact of θ on the prices of goods A and B. Let us rewrite the first order conditions (B1a), (B1b') and (B2) in general notation. Recall that (B1a) is independent of both the price of A and therefore θ . Rewrite (B1b') as the following implicit general function

$$f(P_B, P_A, \theta) = 0 \quad (B1b'')$$

where P_B is implicitly a function of P_A and θ .

To find $\frac{dP_B}{d\theta}$, we totally differentiate (B1b'') to get:

$$\frac{dP_B}{d\theta} = \frac{-\left(\frac{df}{dP_A} \frac{dP_A}{d\theta} + \frac{df}{d\theta}\right)}{\frac{df}{dP_B}}$$

From second order conditions we know that the denominator is negative, and thus

$$\text{Sgn}\left(\frac{dP_B}{d\theta}\right) = \text{sgn}\left(\frac{df}{dP_A} \frac{dP_A}{d\theta} + \frac{df}{d\theta}\right).$$

Solving for each of these derivatives from the actual function f in equation (B1b), we find the following:

$$\frac{df}{d\theta} = P_B f(P_B) \frac{1}{(1+\theta)^2} + \int_{P_A}^{\infty} f(V_A | V_A > P_A) f\left(\frac{P_B - \theta V_A}{1+\theta}\right) \frac{P_B + V_A}{(1+\theta)^2} dV_A > 0$$

and

$$\frac{df}{dP_A} = -f(P_A | P_A > P_A) f\left(\frac{P_B - \theta V_A}{1+\theta}\right) \frac{P_B + V_A}{(1+\theta)^2} < 0$$

However, we need to find $\frac{dP_A}{d\theta}$ which proves much more difficult. To do so, rewrite (B2) implicitly in general form as

$$g(P_A, P_B(P_A, \theta), \theta) = 0 \quad (\text{B2}')$$

Totally differentiating (B2') and solving for $\frac{dP_A}{d\theta}$, we get:

$$\frac{dP_A}{d\theta} = \frac{-\left(\frac{dg}{dP_B} \frac{\delta P_B}{\delta \theta} + \frac{dg}{d\theta}\right)}{\frac{dg}{dP_A} + \frac{dg}{dP_B} \frac{dP_B}{dP_A}}$$

Where the denominator is negative by second order conditions, and thus,

$$\text{Sgn}\left(\frac{dP_A}{d\theta}\right) = \text{sgn}\left(\frac{dg}{dP_B} \frac{dP_B}{d\theta} + \frac{dg}{d\theta}\right)$$

Let us consider the sign of each of these pieces in turn. First note that the direct effect of θ on P_B is $\frac{dP_B}{d\theta} > 0$. This is $\frac{dg}{d\theta}$ from above equation (B2'). The other pieces of $\text{sgn}\left(\frac{dg}{dP_B} \frac{dP_B}{d\theta} + \frac{dg}{d\theta}\right)$ prove more difficult to sign. As with the results in appendix A, the signs will depend upon the signs of f' and f'' which are indeterminate. Moreover, we would need to sign the cross partial $\frac{d^2 P_B}{dP_A d\theta}$. As such, arriving at a sign for $\frac{dP_A}{d\theta}$ is intractable, as is determining the sign for $\frac{dP_B}{d\theta}$ which depends upon the sign of $\frac{dP_A}{d\theta}$.

However, while a solution for the impact of changes in θ on prices proves difficult, the impact on profit can be found. Using the chain rule, we know that

$$\frac{d\Pi^*}{d\theta} = \frac{d\Pi}{d\theta} + \frac{d\Pi}{dP_{B_0}} \frac{dP_{B_0}^*}{d\theta} + \frac{d\Pi}{dP_{B_0}} \frac{dP_{B_0}^*}{dP_A} \frac{dP_A}{d\theta} + \frac{d\Pi}{dP_{B_1}} \frac{dP_{B_1}^*}{d\theta} + \frac{d\Pi}{dP_{B_1}} \frac{dP_{B_1}^*}{dP_A} \frac{dP_A}{d\theta} + \frac{d\Pi}{dP_A} \frac{dP_A^*}{d\theta}$$

Recall that $\frac{dP_{B_0}^*}{d\theta} = 0$ and $\frac{dP_{B_1}^*}{dP_A} = 0$. By the envelope theorem, the last 3 terms = 0 and recall

$$E\Pi(P_A) = F(P_A) P_{B_0}^* \left(1 - F(P_{B_0}^*)\right) + (1 - F(P_A)) P_{B_1}^* \int_{P_A}^{\infty} \left(1 - F\left(\frac{P_{B_1}^* - \theta V_A}{1+\theta}\right)\right) f(V_A | V_A > P_A) dV_A$$

So,

$$\frac{dE\Pi}{d\theta} = (1 - F(P_A))P_{B_1}^* \int_{P_A}^{\infty} f(V_A|V_A > P_A) f\left(\frac{P_{B_1}^* - \theta V_A}{1 + \theta}\right) \left(\frac{P_{B_1}^* + V_A}{(1 + \theta)^2}\right) dV_A > 0$$

since each term is positive. This leads to Proposition 4.

Proposition 4: When a monopolist engages in sequential pricing with discrimination, expected profit is increasing in θ . In other words, as intuition would suggest, as goods become more complementary, sequential pricing results in greater expected profits.

Appendix C. Rational Buyers in a Simple Environment

A buyer is considering purchasing two goods A and B from a monopolist who offers the two goods sequentially and sets the price of good B after observing the buyer's purchase decision for good A. Suppose a buyer is type H (value = V) with probability p and is of type L (value = v) with probability $1 - p$. Assume $pV > v$. A seller chooses prices for each good denoted P_A and P_B and suppose that $P_A = V - \Delta > v$ since no buyer would purchase good A at $P_A = V$ if there was any positive probability that $P_B < V$. This is because in doing so they reveal their type and guarantee that $P_B = v$.

Seller Strategy: If buyer buys at P_A , the seller knows the buyer is type H and sets $P_B = V$ so $\pi = V - \Delta + V$, $CS = V - \Delta$ for type H .

If the buyer does not buy good A, the seller will need to update her belief that the probability the buyer is type H ; the updated probability depends on the rational buyer's strategy. Define this updated probability to be q . $P_B = V$ iff $qV > v$, where q is determined in equilibrium.

Buyer Strategy: If buyers separate: i.e, type H buys at $V - \Delta$ with probability 1, then

$q = 1$ if good A is purchased and $P_B = V$, $CS = V - \Delta$

$q = 0$ if good A is not bought and $P_B = v$, $CS = V - v$ which is greater

\Rightarrow type H buyer will not buy good A and a separating equilibrium cannot be supported.

Mixed Strategy Equilibrium:

If seller observes no purchase, then the seller chooses $P_B = V$ with probability α and $P_B = v$ with probability $1 - \alpha$. A Type H buyer is indifferent between buying at $V - \Delta$ and not iff $\Delta = (1 - \alpha)(V - v)$. This identifies the equilibrium probability for the seller, α^* .

The Seller will be indifferent (and \Rightarrow choose $P_B = V$ with probability α) iff $\beta \equiv$ probability type H buys good A $V - \Delta$ which implies $q = \frac{p(1-\beta)}{p(1-\beta)+1-p}$ is the updated probability buyer is of type H given they did not buy good A. Therefore, if the seller charges $P_B = V$ when they observe that good A is not purchased then $\pi = \frac{p(1-\beta)}{p(1-\beta)+1-p} \cdot V$ and if they charge v they get a sale with probability 1. The seller is indifferent iff $\frac{p(1-\beta)}{p(1-\beta)+1-p} \cdot V = v$ which implicit defines β^* . α^*, β^* are equilibrium mixed strategy probabilities and the seller's profit would be $\pi = p[\beta(V - \Delta + v) + (1 - \beta)[(1 - \alpha)v + \alpha V]] + (1 - p)((1 - \alpha)v)$.